1) **a)** Let $A$ be an $nxn$ matrix. Show that $A$ can be written as the sum of a symmetric and a skew symmetric matrices.

**b)** Suppose $\{u_1, u_2, u_3, u_4, u_5\}$ be a basis for $R^5$. If $c_2, c_2, c_3, c_4, c_5$ are scalars with $c_3 \neq 0$, show that $\{u_1, u_2, c_1u_1 + c_2u_2 + c_3u_3 + c_4u_4 + c_5u_5, u_4, u_5\}$ is also a basis for $R^5$.

2) Let $C[0,1]$ be the vector space of all continuous real valued functions with domain $[0,1]$. Let $\langle f, g \rangle = \int_0^1 f(x)g(x)dx$ be the inner product in $C[0,1]$ where $f$ and $g$ are two functions in $C[0,1]$. Answer the following questions for $f(x) = x$ and $g(x) = \cos x$.

**a)** Find $\langle f, g \rangle$ and $|g|$ where $|.|$ denotes the length induced by this inner product. Show your work.

**b)** Determine the scalar $c$ so that $f-cg$ is orthogonal to $f$. Show all your work.

3) Let $A = \begin{bmatrix} a & a & -3a \\ 0 & 2 & 9 \\ 0 & 1 & 2 \end{bmatrix}$ where $a$ is a real number.

**a)** Find the characteristic polynomial of $A$. (Of course it may depend on $a$). **Show all your work.**

**b)** What must be $a$ if $A$ is not diagonalizable? **Explain your answer.**

4) Let $(D - 1)^2(D^2 + 4)y = x^5e^{x} + 7\sin 2x$ be the given 4-th order nonhomogeneous differential equation with constant coefficients.

**a)** Find the general solution of the corresponding homogeneous differential equation.

**b)** Write down the form of the particular solution $y_p$ but DO NOT evaluate the coefficients.

5) Use the method of variation of parameters to find a particular solution to the differential equation $y'' - 3y' + 2y = \frac{1}{1+e^{-x}}$. **Show all your work.**