1-a) Find the sum of the series \( \sum_{n=2}^{\infty} \frac{1}{(n-1)(n+1)} \).

**Solution:** Letting \( a_n = \frac{1}{(n-1)(n+1)} \) and using partial fractions technique we get

\[
2a_n = \frac{2}{(n-1)(n+1)} = \frac{1}{n-1} - \frac{1}{n+1}.
\]

Adding the terms of the series we find

\[
2a_2 = 1 - \frac{1}{3},
2a_3 = \frac{1}{2} - \frac{1}{4},
2a_4 = \frac{1}{3} - \frac{1}{5},
\]

\[\vdots\]

\[
2a_{n-2} = \frac{1}{n-3} - \frac{1}{n-1},
2a_{n-1} = \frac{1}{n-2} - \frac{1}{n},
2a_n = \frac{1}{n-1} - \frac{1}{n+1}
\]

from which it follows that the partial sum

\[
S_n = a_1 + \cdots + a_n = \frac{1}{2} \left(1 + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}\right)
\]

and after taking the limit as \( n \to \infty \) we find that the sum is

\[
\sum_{n=2}^{\infty} \frac{1}{(n-1)(n+1)} = \frac{1}{2} \left(1 + \frac{1}{2}\right) = \frac{3}{4}.
\]
1-b) Test for convergence $\sum_{n=1}^{\infty} \frac{2^n \ln n}{n!}$.

Solution: Let $a_n = \frac{2^n \ln n}{n!}$. Using the ratio test we find that

$$\frac{a_{n+1}}{a_n} = \frac{2 \ln(n+1)}{(n+1) \ln n}$$

which converges to zero as $n \to \infty$. Hence the series converges by the Ratio Test.

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2-a) Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$.

Solution: Letting $a_n = \frac{1}{n(n+3)}$ and using partial fractions technique we get

$$3a_n = \frac{3}{n(n+3)} = \frac{1}{n} - \frac{1}{n+3}.$$ 

Adding the terms of the series we find

$$3a_1 = 1 - \frac{1}{4},$$

$$3a_2 = \frac{1}{2} - \frac{1}{5},$$

$$3a_3 = \frac{1}{3} - \frac{1}{6},$$

$$3a_4 = \frac{1}{4} - \frac{1}{7},$$

$$\vdots$$

$$3a_{n-3} = \frac{1}{n-3} - \frac{1}{n},$$

$$3a_{n-2} = \frac{1}{n-2} - \frac{1}{n+1},$$

$$3a_{n-1} = \frac{1}{n-1} - \frac{1}{n+2},$$

$$3a_n = \frac{1}{n} - \frac{1}{n+3}$$

from which it follows that the partial sum

$$S_n = a_1 + \cdots + a_n = \frac{1}{3} \left( 1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3} \right)$$

and after taking the limit as $n \to \infty$ we find that the sum is

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)} = \frac{1}{3} \left( 1 + \frac{1}{2} + \frac{1}{3} \right) = \frac{11}{18}.$$
2-b) Test for convergence $\sum_{n=0}^{\infty} \frac{n!}{2^n}$.

Solution: Let $a_n = n!/2^n$. Using the ratio test we find that

$$\frac{a_{n+1}}{a_n} = \frac{n + 1}{2^{n^2+n+1}}$$

which converges to zero as $n \to \infty$. Hence the series converges by the Ratio Test.