1. Given a homogeneous system of spin-\( \frac{1}{2} \) particles interacting through a potential \( V \)

a) show that the expectation value of the Hamiltonian in the noninteracting ground state is

\[
E^{(0)} + E^{(1)} = 2 \sum_{\mathbf{k}} \frac{k^2 \mathbf{\hat{e}^2}}{2m} + \frac{1}{2} \sum_{\mathbf{k}, \mathbf{k}' \lambda, \mathbf{\bar{k}}', \lambda'} \{ <\mathbf{k} \lambda \mathbf{\hat{e}} | V \mathbf{\bar{k}}' \lambda' > <\mathbf{\bar{k}}' \lambda' \mathbf{\hat{e}} | \mathbf{k} \lambda > - <\mathbf{k} \lambda \mathbf{\hat{e}} | V \mathbf{k} \lambda > <\mathbf{\bar{k}}' \lambda' \mathbf{\hat{e}} | \mathbf{\bar{k}}' \lambda' > \}
\]

where \( \lambda \) is the z-component of the spin.

b) Assume \( V \) is central and spin independent. If \( V(\mathbf{i} \mathbf{\hat{z}} - \mathbf{\bar{i}} \mathbf{\hat{z}}) < 0 \) for all \( |\mathbf{i} \mathbf{\hat{z}} - \mathbf{\bar{i}} \mathbf{\hat{z}}| \) and \( \int |V| d^3x < \infty \), prove that the system will collapse.

2. Given a homogeneous system of spin-zero particles interacting through a potential \( V \)

a) show that the expectation value of the Hamiltonian in the noninteracting ground state is

\[
\frac{E^{(1)}}{N} = (N-1) V(0) \frac{1}{2} \leq \frac{1}{2} N V(0) \quad \text{where} \quad V(q) = \int d^3x V(x) e^{-iq \cdot x}
\]

b) Repeat 1-b

c) show that the second-order contribution to the ground-state energy is

\[
\frac{E^{(2)}}{N} = - \frac{N-1}{2V} \int \frac{d^3q}{(2\pi)^3} \frac{|V(q)|^2}{k^2q^2/m} \leq - \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \frac{|V(q)|^2}{k^2q^2/m}
\]