Solution 5.1

a) 
\[(1 + i)^3 = (1 + i)(1 + 2i + i^2) = (1 + i)(1 + 2i - 1) = (1 + i)2i = -2 + 2i\]  \(\text{(1)}\)

b) 
\[\frac{1 + i}{1 - i} = \frac{(1 + i)(1 + i)}{(1 - i)(1 + i)} = \frac{2i}{2} = i\]  \(\text{(2)}\)

c) 
\[e^{2 + i\pi/4} = e^2 \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) = e^2 \frac{\sqrt{2}}{2} (1 + i)\]  \(\text{(3)}\)

d) 
\[\sin\left(\frac{\pi}{4} + 2i\right) = \sin(\pi/4) \cos 2i + \cos(\pi/4) \sin 2i = \frac{\sqrt{2}}{2} (\cos 2i + \sin 2i) = \frac{\sqrt{2}}{2} \left(\frac{e^{-2i^2} + e^{2i^2}}{2} + \frac{e^{2i^2} - e^{-2i^2}}{2i}\right) = \frac{\sqrt{2}}{4} (e^2 + e^{-2} + i(e^2 - e^{-2}))\]  \(\text{(4)}\)

Solution 5.2

\[1 = \cos 2\pi k + i \sin 2\pi k\]  \(\text{(5)}\)

where \(k\) is an integer. Then;
\[\sqrt{n} = (e^{i2\pi k})^{1/n}\]  \(\text{(6)}\)

or
\[\sqrt{n} = (e^{i2\pi/n})^k\]  \(\text{(7)}\)
We see that \(1^{(1/n)}\) has \(n\) complex roots in the form \((W_n)^k\) such that:
\[
W_n = \cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n}
\]
where
\[
k = 0, 1, 2, ..., n - 1
\]
When \(k \geq n\) we don’t get any different root, that is the root is already contained in the set \(k = 0, 1, 2, ..., n - 1\) as you can easily prove by using Eq.7. So the roots of \(1^{(1/n)}\) are equally spaced around the unit circle in the complex plane.

**Solution 5.3**

\[
|R_1 e^{i\theta_1} + R_2 e^{i\theta_2}|^2 = |R_1 (\cos \theta_1 + i \sin \theta_1) + R_2 (\cos \theta_2 + i \sin \theta_2)|^2
= |(R_1 \cos \theta_1 + R_2 \cos \theta_2) + i(R_1 \sin \theta_1 + R_2 \sin \theta_2)|^2
= (R_1 \cos \theta_1 + R_2 \cos \theta_2)^2 + (R_1 \sin \theta_1 + R_2 \sin \theta_2)^2
= R_1^2 \cos^2 \theta_1 + R_2^2 \cos^2 \theta_2 + 2R_1 R_2 \cos \theta_1 \cos \theta_2
+ R_1^2 \sin^2 \theta_1 + R_2^2 \sin^2 \theta_2 + 2R_1 R_2 \sin \theta_1 \sin \theta_2
= R_1^2 + R_2^2 + 2R_1 R_2 (\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2)
= R_1^2 + R_2^2 + 2R_1 R_2 \cos(\theta_1 - \theta_2)
\]

**Solution 5.4**

Let me denote the miss distance as \(\Delta l\) and uncertainty in position as \(\Delta x\). We are searching for the minimum uncertainty, so that the uncertainty relation becomes:
\[
\Delta p \Delta x = \hbar/2
\]
Note that \(\Delta x = 2\Delta l\), \(\Delta p = m \Delta v\), \(\Delta l = t \Delta v\) and \(t = \sqrt{\frac{2H}{g}}\). Insert all these equations into Eq.11;
\[
2m \frac{\Delta l}{\sqrt{\frac{2H}{g}}} \Delta l = \frac{\hbar}{2}
\]
Then the miss distance \(\Delta l\) will be;
\[
\Delta l = \left(\frac{\hbar}{4m}\right)^{1/2} \left(\frac{2H}{g}\right)^{1/4}
\]
or
\[ \Delta l = \left( \frac{\hbar}{2m} \right)^{1/2} \left( \frac{H}{2g} \right)^{1/4} \]  
\tag{14}

b)
\[ \Delta l = \left( \frac{6.63 \times 10^{-34}}{4\pi \times 0.5 \times 10^{-3}} \right)^{1/2} \left( \frac{2.0}{2 \times 9.8} \right)^{1/4} \approx 1.8 \times 10^{-16} \text{ m} \]  
\tag{15}

Solution 5.5

a)
\[ h = 2\pi \text{ J.s} \Rightarrow \hbar = 1 \text{ J.s} \]  
\tag{16}

The uncertainty relation becomes;
\[ \Delta p \Delta x \geq \frac{1}{2} \text{ J.s} \]  
\tag{17}

We are searching for the minimum uncertainty, that is;
\[ \Delta p \Delta x = \frac{1}{2} \text{ J.s} \]  
\tag{18}

Using \( \Delta p = m \Delta v \) and \( \Delta x = 1 \text{ m} \);
\[ m \Delta v \Delta x = \frac{1}{2} \Rightarrow \Delta v = \frac{1 \text{ J.s}}{2 \times 2.0 \text{ kg} \times 1.0 \text{ m}} = 0.25 \text{ m/s} \]  
\tag{19}

b)

The uncertainty in position increases with time (\( \Delta x = t \Delta v \)) such that after 5.0 s it increases \( 2 \times 0.25 \text{ m/s} \times 5.0 \text{ s} = 2.5 \text{ m} \), then with the initial uncertainty;
\[ \Delta x = 3.5 \text{ m} \]  
\tag{20}