Solution 4.1 Problem 10 of Ch.5

The diameter of the hydrogen atom is about $10^{-10}$ m.

$$\lambda = \frac{h}{p} \quad \text{and} \quad K = \frac{p^2}{2m} \quad (1)$$

Then;

$$K = \frac{1}{2m} \left( \frac{h}{\lambda} \right)^2 \quad (2)$$

$$= \frac{1}{2 \times 9.1 \times 10^{-31} \text{kg}} \left( \frac{6.63 \times 10^{-34} \text{J.s}}{10^{-10} \text{m}} \right) \approx 2.41 \times 10^{-17} \text{J}$$

Solution 4.1 Problem 11 of Ch.5

Using Eq.2;

$$K = \frac{1}{2m} \left( \frac{h}{\lambda} \right)^2 \quad (3)$$

$$= \frac{1}{2 \times 9.1 \times 10^{-31} \text{kg}} \left( \frac{6.63 \times 10^{-34} \text{J.s}}{10^{-14} \text{m}} \right) \approx 2.41 \times 10^{-9} \text{J}$$

Then the velocity of the electron will be larger than the speed of light. So it can’t.

Solution 4.2 Problem 37 of Ch.4

According to Bohr model the energy of the $n^{th}$ state of the Hydrogen atom is given as;

$$E = -\frac{me^4k^2}{2\hbar^2} \frac{1}{n^2} \quad (4)$$

Then the frequency of the emitted light due to transition from the state $n$ to state $n-1$ is;

$$hf = -\frac{me^4k^2}{2\hbar^2} \left( \frac{1}{n^2} - \frac{1}{(n-1)^2} \right) \quad (5)$$

or;

$$f = -\frac{me^4k^2}{2\hbar^2} \left[ \frac{-2n+1}{n^2(n-1)^2} \right]$$
\[ f = \frac{4\pi^2 mk^2 e^4}{h^3} \frac{1}{n^3} \]  \tag{7}

The classical frequency is:

\[ f = \frac{\nu}{2\pi r} = \frac{k e^2}{\hbar} \frac{2\pi \hbar^2}{k m e^2} = \frac{4\pi^2 mk^2 e^4}{h^3} \frac{1}{n^3} \]  \tag{8}

which as same as Eq.7.

**Solution 4.2 Problem 43 of Ch.4**

The change in momentum of atom in the beam is \( \frac{h}{\lambda} \):

\[ \Delta p = m \Delta v = \frac{h}{\lambda} \]  \tag{9}

The average deceleration is then:

\[ \frac{\Delta v}{\Delta t} = \frac{h}{m \lambda \Delta t} \approx 1.3 \times 10^6 \text{ m/s}^2 \]  \tag{10}

The distance over which the atoms will be brought to halt is:

\[ x = \frac{1}{2} \frac{10^6 \times 10^3}{10^6} \approx 0.5 \text{ m} \]  \tag{11}

**Solution 4.3**

If you figure out \( f(x) \) you will see that the period of \( f(x) \) is \( 2\pi \). Or;

\[ [4(n + 1) - 1] \frac{\pi}{2} - [4n - 1] \frac{\pi}{2} = 2\pi \]  \tag{12}

b) First of all I should note that \( f(x) \) is an even function. You can easily check that \( f(-x) = f(x) \). The Fourier series of a function \( f(x) \)
is given as;
\[ f(x) = \frac{b_0}{2} + \sum_{k=1}^{\infty} b_k \cos(kx) + \sum_{k=1}^{\infty} a_k \sin(kx) \quad (13) \]

where
\[
\begin{align*}
  b_0 &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)dx \\  b_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(kx)dx  \\  a_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(kx)dx
\end{align*}
\]

Sine is an odd function so that the value of the integral in Eq.16 is zero for all \(k\). So we don’t have \(\sin(kx)\) terms in the series.

Let’s evaluate Eq.14;
\[
\begin{align*}
  b_0 &= \frac{1}{\pi} \left[ -\int_{-\pi/2}^{\pi/2} f(x)dx + \int_{-\pi/2}^{\pi/2} f(x)dx - \int_{\pi/2}^{\pi} f(x)dx \right] \\
  &= \frac{1}{\pi} \left[ -(-\pi + \pi) + (\pi + \pi) - (\pi - \pi) \right] = 0
\end{align*}
\]

Finally let’s find the coefficients \(b_k\);
\[
\begin{align*}
  b_k &= \frac{1}{\pi} \left[ -\int_{-\pi/2}^{\pi/2} \cos(kx)dx + \int_{-\pi/2}^{\pi/2} \cos(kx)dx - \int_{\pi/2}^{\pi} \cos(kx)dx \right] \\
  &= \frac{1}{\pi k} \left[ -\sin(kx)|_{-\pi/2}^{\pi/2} + \sin(kx)|_{-\pi/2}^{\pi/2} - \sin(kx)|_{\pi/2}^{\pi} \right] \\
  &= \frac{1}{\pi k} \left[ -2\sin(\frac{k\pi}{2}) + 2\sin(\frac{k\pi}{2}) \right]
\end{align*}
\]

So;
\[
\begin{align*}
  b_k &= 0 \quad & k &= 2l \\  b_k &= \frac{4}{k\pi} \quad & \text{if } k &= 4l + 1 \\  b_k &= -\frac{4}{k\pi} \quad & \text{if } k &= 4l + 3
\end{align*}
\]

where \(l\) is an any integer.