The exam consists of 5 questions of equal weight.

Please read the questions carefully.

Show all your work in legibly written, well-organized mathematical sentences.

Calculators and dictionaries are not allowed.

Simplify as far as possible unless otherwise stated.

Please turn off your cellular phones before the exam starts.

Extra sheets of paper, if needed, will be provided.
1) a) Evaluate \( \lim_{x \to 0} \frac{1}{\sin(x)} \int_0^{\sin(2x)} \cos(5t) \, dt \)

b) A hallway of width 4 ft meets a hallway of width \( 12\sqrt{3} \) ft at a right angle. Find the length of the longest ladder that can be carried horizontally around the corner from one hallway to the other hallway. (i. While solving this problem, to find the global extrema of a function that is not defined on a closed interval you have to use the first derivative test or the second derivative test.) (ii. Find the exact answer.)
2) Define a function $f$ as follows

$$ f(x) = \frac{x^2}{x^2 - 4}. $$

You have to explicitly write the answer to each part.

a) Find the domain of the function $f$.

b) Find all asymptotes, the $x$-intercept(s) and the $y$-intercept(s) of the graph of $f$.

c) Find $f'(x)$ and $f''(x)$. Find

d) Find the intervals on which the function $f$ is increasing and decreasing. Identify the function’s local extreme values, if any, saying where they are taken on.

e) Find where the graph of $f$ is concave up and where it is concave down. Are there any inflection points on the graph of $f$?

f) Sketch the graph of $f$. 
3) a) Calculate the following limit \( \lim_{n \to \infty} \frac{1}{n^3} \sum_{i=1}^{n} i^2 \sin\left(5 + \frac{\pi i^3}{n^3}\right) \).

b) Calculate \( \int \frac{\sin(2x)}{3 + 2 \cos^2(x)} \, dx \).
4) Let $f$ be a continuous function on $[2, 5]$. Assume that

$$f(7 - x) = f(x) \neq 0$$

for all $x$ in $[2, 5]$ and

$$\int_{4}^{5} f(u)du = 26.$$ 

If we define

$$F(x) = \int_{4}^{x} f(u) du \quad \text{and} \quad G(x) = \int_{3}^{x} f(u) du$$

for all $x$ in $[2, 5]$.

a) Show that $G$ is an increasing function on $[2, 5]$.

b) Show that there exists only one number $c$ in the interval $[2, 5]$ such that $F(c) = 0$.

c) Calculate $F(2) + G(4)$. 
5) a) Find the volume of the solid generated by revolving the region bounded by the curves $x = \ln(y)$, $y = e^{2x}$, and $x = \ln(2)$ about the $x$-axis.

b) Find the surface area of the solid generated by revolving the curve $y = \sqrt{2x - x^2}$ for $1 \leq x \leq \frac{3}{2}$.