MATH 132 (Section 3) FINAL EXAM

IMPORTANT
1. This exam consists of 5 questions of equal weight.
2. Each question is on a separate sheet. Please read the questions carefully and write your answers under the corresponding questions. Be neat.
3. Show all your work. Correct answers without sufficient explanation might not get full credit.
4. Calculators are not allowed.

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1. a) Prove that given any 16 integers from 1 to 30, some three of them have an odd sum.

b) In how many ways can the 26 letters of the alphabet be permuted so that non of the patterns exam, think, way and dog occurs?

[10 + 10 points]
2. Find the number of sequences $x_1, x_2, \ldots, x_{2006}$ for which $x_i = 1, 2, 3$ or 4 and the total number of 3 entries is even.

[20 points]
3. Let $A = \{a, b, c, d, e, f, g, h, i\}$. Find the number of equivalence relations on $A$ such that $(a, b)$ and $(a, c)$ are not related.

[20 points]
4. a) Let \( G \) be a directed complete graph with 2006 vertices. For each vertex \( v \) the incoming degree is denoted by \( id(v) \) and the outgoing degree is denoted by \( od(v) \). Prove that \( \sum_v (id(v))^2 = \sum_v (od(v))^2 \).

b) Prove or disprove that that the following graph has a Hamiltonian path:
5. Let $K_{12}$ be a complete graph with 12 vertices, $G$ be a subgraph of $K_{12}$ and $G'$ be its complement.

a) Prove or disprove that either $G$ or $G'$ is planar.

b) Suppose that $G$ is planar. Prove or disprove that $G'$ is not planar.

[10 + 10 points]