MATH 132 (Section 3) MIDTERM 2 EXAM

IMPORTANT
1. This exam consists of 5 questions of equal weight.
2. Each question is on a separate sheet. Please read the questions carefully and write your answers under the corresponding questions. Be neat.
3. Show all your work. Correct answers without sufficient explanation might not get full credit.
4. Calculators are not allowed.

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1. a) How many ways are there to distribute six different toys to four different children such that exactly one child gets no toy?

b) Let $B$ be a set \{a, b, c, d, e, f, g, h, i\}. Find the number of total orders $\mathcal{R}$ on $B$ such that $a \mathcal{R} f$ and $f \mathcal{R} h$.

[10 + 10 points]
2. \( A = \{1, 2, \ldots, 9\} \), find the number of equivalence relations on \( A \) such that 1 and 2 are related but 7 and 8 are not related.

[20 points]
3. a) Prove that at a party where there are 2006 people, there are two people who know the same number of other people there.

b) Show that the absolute value of the difference between \( k \sqrt{2} \) and the nearest integer to \( k \sqrt{2} \) is less than \( \frac{1}{n} \) for some positive integer \( k \) not exceeding \( n \).

[10 + 10 points]
4. Determine the number of nonnegative integer solutions of

\[ 0 < x_1 + x_2 + x_3 + x_4 < 50 \]

where \( x_1 \leq 13, \ x_2 \leq 13, \ x_3 \leq 13, \ x_4 \leq 20 \)
5. In how ways are there to tile (to cover without overlapping) $2 \times n$ rectangle with $2 \times 2$ square, $2 \times 1$ vertical and $1 \times 2$ horizontal rectangles?

[20 points]