MATH 132 (Section 3) MIDTERM 1 EXAM

IMPORTANT
1. This exam consists of 5 questions of equal weight.
2. Each question is on a separate sheet. Please read the questions carefully and write your answers under the corresponding questions. Be neat.
3. Show all your work. Correct answers without sufficient explanation might not get full credit.
4. Calculators are not allowed.

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1. a) In how many ways can 9 boys and 9 girls sit in a row, if no two people of the same sex are allowed to sit together?

b) In how many ways can a man divide 7 different gifts among his 3 children if one child is to receive 3 gifts and the others 2 each?

[10 + 10 points]
2. Determine the number of integer solutions of for

\[ 0 < x_1 + x_2 + x_3 + x_4 + x_5 + x_6 < 50 \]

where \( x_i \geq -2 \) [20 points]
3. a) If $p$ and $r$ are primitive statements, prove that

b) Prove that each natural number $n > 23$ can be written as a sum of 5's and 7's.

[10 + 10 points]
4. a) Let $a$ and $b$ be odd natural numbers. Prove that $a^2 + b^2$ is not a square of any natural number.

b) Let $p$ be a prime number exceeding 3. Prove that 24 divides $p^2 - 1$.

[10 + 10 points]
5. a) Let \( n \) be a natural number. What possible values can \( \gcd(5n + 3, 7n+4) \) have?

b) For natural number \( n \) define \( X_n = \{1, 2, \ldots, n\} \). A function \( f \) is called a monotone decreasing function, if for all \( i < j \) we have \( f(i) > f(j) \). Find the number of monotone decreasing functions \( f : X_{10} \to X_{30} \).

[10 + 10 points]