MATH 116
HOMEWORK I

Problem 1. Let $f$ be the function defined by $f(x, y) = \frac{y^2 - x^2}{x + 2y^3}$.

(a) Use the $\varepsilon - \delta$ definition of limit to verify that $\lim_{(x,y) \to (2,2)} f(x, y) = 0$.

(b) Find the limit $\lim_{(x,y) \to (0,0)} f(x, y)$, or show that the limit does not exist.

Problem 2. Consider the function

$$z = f(y, xg(x - y, y - x)).$$

Find $\frac{\partial^2 z}{\partial x \partial y}(1, 1)$, if

$g(1, 1) = 0$, \hspace{0.5cm} g(0, 0) = f_2(1, 2) = 2,$

$g_2(0, 0) = f_1(1, 1) = g_1(1, 1) = 0$, \hspace{0.5cm} $f_1(1, 2) = g_1(0, 0) = f_{22}(1, 2) = 3,$

$f_{12}(1, 2) = f_{21}(1, 2) = -2$, \hspace{0.5cm} $f_{11}(1, 2) = g_{11}(0, 0) = -4,$

$g_{12}(0, 0) = f_{22}(1, 1) = g_{22}(0, 0) = g_{21}(0, 0) = g_{22}(1, 2) = -1.$

Problem 3. Find $\nabla f(a, b)$ for the differentiable function $f(x, y)$ given the directional derivatives

$$(D_{\vec{u}}f)(a, b) = 3\sqrt{2} \quad \text{and} \quad (D_{\vec{v}}f)(a, b) = 5,$$

where

$$\vec{u} = \frac{i + j}{\sqrt{2}} \quad \text{and} \quad \vec{v} = \frac{3i - 4j}{5}.$$

Problem 4. Find and classify the critical points of the function $f(x, y) = x^2 + y^2 + \frac{32}{xy}$.

Problem 5. Find the points of the sphere $x^2 + y^2 + z^2 = 9$ that are closest to $(10, 4, -5)$. 