MATH 116 INTERMEDIATE CALCULUS III
FINAL EXAM
Date: July 22, 2005, Time: 9:00-11:00

SURNAME/NAME:...................................................

ID:................................... Section.................

1 Check that there are 5 questions on your booklet.
2 Show all your work. Correct answers without sufficient explanation may not get full credit.

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Problem 1. Suppose that function $f(x, y, z)$ has continuous second order partial derivatives and $f(x, y, z) = g(r)$, where $r = \sqrt{x^2 + y^2 + z^2}$. Evaluate $f_{xx} + f_{yy} + f_{zz}$ at point $(2, -2, 1)$ given that $g_r(3) = 6$ and $g_{rr}(3) = 1$. 
Problem 2. Show that the curve \( \vec{r}(t) = \sqrt{t} \, \vec{i} + \sqrt{t} \, \vec{j} - \frac{t + 3}{4} \, \vec{k} \) is normal to the surface \( x^2 + y^2 - z = 3 \) at the intersection point.
Problem 3. Let \( F = (3x^2y + z^2)\vec{i} + (x^3 - 2yz)\vec{j} + (2xz - y^2)\vec{k} \) be a vector field.

(a) Show that vector field \( F \) is conservative.

(b) Evaluate the counterclockwise circulation \( \oint_C F \circ dr \), where \( C \) is the graph of function
\[
\vec{r}(t) = \sin \frac{t}{2}\vec{i} + \tan \frac{t}{4}\vec{j} + \frac{t}{\pi}\vec{k}, \quad 0 \leq t \leq \pi.
\]
Problem 4. Let $C$ be the boundary of the rectangle having vertices at points $(0,0,0)$, $(0,3,3)$, $(1,3,3)$, $(1,0,0)$ oriented in the clockwise direction when viewed from high on the $z$-axis. Find circulation $\oint_C F \cdot dr$ of the vector field $F = x^2 \vec{i} + 4xy^3 \vec{j} + y^2 x \vec{k}$ around the curve $C$. 
Problem 5. Let $D$ be the region given by $x^2 + y^2 + z^2 \leq 4a^2$ and $x^2 + y^2 \geq a^2$. Let $S$ be the surface of solid $D$. Evaluate the flux $\int_S F \cdot \mathbf{n} \, d\sigma$ of the vector field $F = (x + yz)\mathbf{i} + (y - xz)\mathbf{j} + (z - e^x \sin y)\mathbf{k}$ across the surface $S$. 