MATH 215 Homework 2

Turn in by June 20, 2005 until 9:40 a.m.

Each question is 4 points, total: 20 points.

In problems 1, 2, and 3, $X$ will denote an arbitrary metric space with the metric $d$.

1. Let $E \subset X$. Show that $\text{int} E$ is an open set.

2. If $A$ is an open set and $A \subset E \subset X$, then $A \subset \text{int} E$.

3. a) Let $\{E_i : i \in I\}$ be any collection of open sets in $X$. Show that $\bigcup_{i \in I} E_i$ is also open.
   b) Let $E_1, E_2, \ldots, E_n$ be a finite collection of open sets in $X$. Show that $E_1 \cap E_2 \cap \cdots \cap E_n$ is also open.

4. For the following set $E \subset \mathbb{R}^2$, draw the picture of the set and find $\text{int} E$, $E'$ and $\overline{E}$. Determine also whether $E$ is connected. No proofs are needed.
   For $n \in \mathbb{N}$, let $E_n = \{(x, y) : x = \frac{1}{n}, y \in \mathbb{R}, 0 < y \leq \frac{1}{n}\}$, and $E = \bigcup_{n=1}^{\infty} E_n$.

5. By using only the definition, show that the set $E$ in Problem 4 is not compact, i.e., find an open cover of $E$ which does not have a finite subcover.