Instructions

Please read the questions carefully before writing your solutions.
Use separate sheets for different questions.
Make sure your handwriting is legible.
Make sure that your line of reasoning for a given problem is clearly reflected in your solution.
You may lose points if the above guidelines are not satisfactorily followed.
Let me remind you that cheating is a capital offense. Disciplinary action will be taken if the confidence entrusted in you is breached.
1) Transform the second order ODE
\[ y'' + P(x)y' + Q(x)y = 0 \]
by the substitution
\[ y = ze^{-\int P(t)dt} \]
to find the resulting differential equation for \( z \).

2) Consider the following ODE
\[ (1 - x^2)U_n''(x) - 3xU_n'(x) + n(n + 2)U_n(x) = 0 \]
a) Locate the singular points and show whether they are regular or irregular.
b) Put this equation in self-adjoint form.
c) Identify the eigenvalue.
d) Identify the weight function.

3) One solution of Laguerre’s differential equation
\[ xy'' + (1 - x)y' + ny = 0 \]
for \( n = 0 \) is \( y_1(x) = 1 \). Develop a second, linearly independent solution.

4) Construct the Green’s function for the operator \( d^2/dx^2 \) and the boundary conditions \( y(0) = 0 \) and \( y'(1) = 0 \).
Consider now the equation
\[ y''(x) + \lambda y(x) = 0 \]
with the same boundary conditions. Obtain the solutions.
Verify that the solutions satisfy
\[ y(x) = \lambda \int_0^1 G(x, t)y(t) \, dt \]

Good Luck