1. For any positive integer $n$ determine
\[ \sum_{i=0}^{n} \frac{(-1)^n}{i!(n-i)!} \]

2. In how many ways can a man divide 7 different gifts among his 3 children if one child is to receive 3 gifts and the others 2 each?

3. Let $n$ be a natural number. What possible values can $\gcd(5n+3, 7n+4)$ have?

4. Using the principle of mathematical induction prove that $2^{2n+1} + 1$ is divisible by 3 for all integers $n \geq 0$.

5. Find a combinatorial argument to show that the following equality holds for all $n \geq 0$
\[ \binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}. \]

6. A “codeword” from the alphabet \{0,1,2,3\} is said to be legitimate if it contains an even number of zeros. Thus, for instance, the codeword 31010 is legitimate whereas 01010 is not. How many $n$-letter codewords are legitimate?

7. Determine the number of integer solutions for
\[ x_1 + x_2 + x_3 + x_4 < 20 \]
where $x_i \geq 0$ for all $i$.

8. Simplify the following logical statement:
\[ p \land (p \Rightarrow q) \land \neg r \lor q \]

9. Prove that if we select 101 integers from the set $S = \{1, 2, 3, \ldots, 200\}$, there exists $m, n$ in the selection where $\gcd(m, n) = 1$. 


10. A mathematics professor has eight different math books on a bookshelf. Five of these books are on algebra and the other three are on analysis. In how many ways can the professor arrange these books on the shelf if all the algebra books must be next to each other?

11. In how many different ways can we distribute $n$ identical pennies to $k$ children so that each child gets at least one penny?

12. What is the following sum?

$$A = \sum_{k=1}^{n} k \binom{n}{k}.$$

13. Alice has 10 balls (all different). First, she splits them into two piles; then she picks one of the piles with at least two elements, and then splits it into two; she repeats this until each pile has only one element. Find the number of different ways in which she can carry out this procedure.

14. Find the number of arrangements of all the letters in MASSASAUGA in which all three $S$’s are together.

15. In how many ways can 10 (identical) candy bars be distributed among five children so that the youngest gets only one or two of them?

16. Verify that the statement

$$[p \land (p \Rightarrow q)] \Rightarrow q$$

is a tautology using a truth table.

17. Let $A, B$ be sets with $|A| = 10$, $|B| = 18$. What is the total number of relations one can define from $A$ to $B$?

18. Prove or disprove: $2 + 8 + 18 + \cdots + 2n^2 = n^2 + n$.

19. Find the least $n$ for which the statement is true and then prove that $10n < 3^n$.

20. An executive buys $2490 worth of presents for the children of her employees. For each girl she gets an art kit costing $33; each boy receives a set of tools costing $29. How many presents of each type did she buy?