Surfaces of Revolution. Let $\alpha(t) = (g(t), h(t), 0)$ be a plane curve in $\mathbb{R}^3$ (profile curve) where $t \in I$. Rotating this curve about the $x-$axis we obtain a surface $M$ (surface of revolution) with the parametrization

$$X(t, v) = (g(t), h(t) \cos v, h(t) \sin v)$$

with $h(t) > 0$, where $(t, v) \in D = \{t \in I, 0 < v < 2\pi\}$. The coordinate curves are the meridians defined by $\beta(t) = X(t, v_0)$ and are the parallels defined by $\gamma(v) = X(t_0, v)$ where $t_0 \in I$ and $v_0 \in (0, 2\pi)$ are some fixed real numbers.

QUESTIONS

1. Find the coefficients of the first and the second fundamental forms of $M$.
2. Prove that both of the coordinate curves are principal curves of $M$.
3. Under what conditions the coordinate curves are asymptotic curves of $M$.
4. Under what conditions the coordinate curves are geodesics of $M$.
5. Find the principal, the Gaussian and the mean curvatures of $M$.
6. Find all surfaces of revolutions with zero Gaussian curvature.