MATH 337
INTRODUCTION TO SOLITON THEORY
Second Midterm Exam

March 20, 2012
Thursday 10.40-12.30, SAZ-20

QUESTIONS.

[35]1. Write a functional, $J[u] = \int_{-\infty}^{\infty} Ldx$, so that Euler-Lagrange equation, $E(L) = 0$, is the modified KdV equation.

[35]2. Hirota bilinear form of the KdV equation, $u_t - 6uu_x + u_{xxx} = 0$, is $(D_t D_x + D^3)(f \cdot f) = 0$ where $u = -2 \partial^2 \ln f / \partial x^2$. Here $D_t$ and $D_x$ are the Hirota differential operators. Find two soliton solutions of the KdV equation.

[35]3. Let $u$ satisfy the Sine-Gordon equation $u_{xt} = \sin u$. Show that the relations below among $u$ and $v$ are the auto Backlund transformations

$$(u + v)_x = 2\lambda \sin \frac{u - v}{2}$$

$$(u - v)_t = \frac{2}{\lambda} \sin \frac{u + v}{2}$$

for the sine-Gordon equation where $v$ also satisfies the Backlund transformation and $\lambda$ is the Backlund parameter. Find a solution for $u$ when $v = 0$. 

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