Math 101
First Midterm Exam
March 12, 2011
12:30 - 14:30

Name : ____________________________________________
ID# : ____________________________________________
Department : _____________________________________
Section : _________________________________________

• The exam consists of 5 questions of equal weight.

• Read the questions carefully.

• Solutions, not answers, get points. Show all your work in well-organized mathematical sentences. Explain your reasoning carefully and in full.

• What can not be read will not be read. Write clearly and cleanly.

• Simplify your answers as far as possible unless otherwise stated.

• Calculators and dictionaries are not allowed.

• Turn off all electronic devices including your cell phones before the exam starts.

Please do not write below this line.

<table>
<thead>
<tr>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>100</td>
</tr>
</tbody>
</table>
1a. Show that

\[
\lim_{x \to 0^+} \sqrt{x} e^{\cos \frac{x}{x}} = 0
\]

1b. Find the limit

\[
\lim_{x \to +\infty} \left[ x \sqrt{3x^2 + 4e^{-x} + 1} - x \sqrt{3x^2 + 2e^{-x}} \right]
\]
2a. The following function is differentiable everywhere

\[ f(x) = \begin{cases} 
\sin(ax) + b, & x < 0 \\
\sin^2(2x) + 2x, & x \geq 0
\end{cases} \]

Find the constants \( a \) and \( b \).

2b. Find the derivative of the function \( y = 2(\ln x)^{x/2} \).
3a. Find the tangent and the normal lines to the curve $x^2 + y^2 = y^4 + 1$ at the point (1,1).

3b. Suppose that the differentiable function $y = f(x)$ has an inverse and that the graph of $f(x)$ passes through the point (1,4) with slope 2. Find the slope of the graph of $f^{-1}(x)$ at (4,1).
4a. Find the absolute maximum and absolute minimum values of the function

\[ f(x) = \frac{x + 1}{x^2 + 2x + 2} \]

on the segment [-4,1].

4b. Show that the function \( f(x) = x^3 + \frac{4}{x^2} + 7 \) has exactly one zero in the interval \((-\infty, 0)\).
5a. Sand falls from a conveyor belt at the rate of 10 m$^3$/min onto the top of a conical pile. The height of the pile is always three-eights of the base diameter. How fast are the (a) height and (b) radius changing when the pile is 4 m high?

5b. Show that the function $y = 2 \sin(\ln x)$ satisfies the equation $x^2 y'' + xy' + y = 0$. 