This is about Archimedes' computation of the area of a segment of a parabola.

(1) Proof Archimedes's result about the area of segments of a parabola using calculus: consider the parabola $y = ax^2$ and compute the area $A$ between the chord defined by $R = (r, ar^2)$ and $S = (s, as^2)$ as well as the area $B$ of the triangle $RST$, where $T = \left( \frac{1}{2}(r + s), \frac{a}{4}(r + s)^2 \right)$, and show that $A = \frac{4}{3}B$.

(2) Compute the area $A$ of a segment of the unit circle defined by the points $R = (-u, \sqrt{1-u^2})$ and $S = (u, \sqrt{1-u^2})$, and compare it to the area $B$ of the triangle with maximal area inside this segment, namely $RST$ with $T = (0, 1)$. Show that the ratio $A : B$ depends on $u$; also derive from your calculations that $\sin x \leq x$ for $x > 0$.

The Chinese had a formula for the area of a segment with base $s = 2u$ and height $h$: $A = \frac{1}{2}(sh + h^2)$. The same formula appears in a manuscript from Cairo written in the third century BC, and in the metrical of Heron of Alexandria. Develop your formula for $A$ into a power series and show that $A = \frac{4}{3}sh + O(u^4)$, where $O(u^4)$ is a power series consisting of terms of order 4 and higher.

(3) We have seen that the triangle Archimedes inscribes into a segment of a parabola is the one with maximal area. Describe the quadrilateral with maximal area that can be inscribed into a segment of a parabola.

Extra credit: compute the ratio of the areas of the segment and the quadrilateral with maximal area.

(4) Reading assignment: read the three articles about zero on the web page.

• Briefly summarize the main claims made by the authors;
• Discuss whether the authors seem biased, and support your claims.