Quiz 06

Name

(1) The graph of \( y = x^2 \) between \( x = 0 \) and \( x = 2 \) is rotated around
(a) the \( x \)-axis;
(b) the \( y \)-axis.
Compute the volumes of the resulting solids.

a) Using cross sections, the volume is

\[
V = \pi \int_0^2 x^4 \, dx = \frac{32}{5} \pi.
\]

Using shells, we integrate in the \( y \)-direction. The radius is just \( y \), the height is \( 2 - x = 2 - \sqrt{y} \), hence

\[
V = 2\pi \int_0^4 y(2 - \sqrt{y}) \, dy = \frac{32}{5} \pi.
\]

b) Using shells, the radius is \( x \), the height is \( y = x^2 \), so the volume is

\[
V = 2\pi \int_0^2 x^3 \, dx = 8\pi.
\]

Using cross sections, you can subtract the volume of the paraboloid from the volume of the big cylinder, which is 16\( \pi \). The volume of the paraboloid is \( \pi \int_0^4 y \, dy = 8\pi \), hence \( V = 16\pi - 8\pi = 8\pi \).