1: Find the generating function for the recurrence relation

\[ a_{n+2} = a_{n+1} + a_n + \frac{1}{n!} \]

with initial conditions \( a_0 = a_1 = 0 \). (Express your answer in a simple form, without any infinite sums. You are NOT required to solve the recurrence relation.)

2: For each integer \( n \geq 0 \), let \( s_n \) be the number of \( n \)-digit sequences where each digit is 0 or 1 or 2, and there are no consecutive 0 digits. (For example, when \( n = 2 \), there are eight possible sequences: 01, 02, 10, 11, 12, 20, 21, 22.) Find a formula for \( s_n \) in terms of \( n \).

3: For an integer \( n \geq 2 \), the graph \( K_n \) is the graph with \( n \) vertices where any two distinct vertices are connected by an edge.

(a) For which values of \( n \) does \( K_n \) have an Euler circuit?

(b) For which values of \( n \) does \( K_n \) have an Euler path which is not an Euler circuit.

(c) Repeat the question for the graph that is obtained from \( K_n \) by deleting one edge.

4: Show that, for any graph with at least two vertices, there exist two different vertices \( x \) and \( y \) which have the same degree as each other.

5: Let \( G \) be a connected planar graph with \( v \) vertices. Let \( d \) be a positive integer and suppose that every vertex of \( G \) has degree \( d \). Show that \( v \) and \( d \) cannot both be odd.