Problem: Let $a, b, c$ be nonnegative real numbers satisfying $a^2 + b^2 + c^2 = 1$. Prove that
\[
\sqrt{a+b} + \sqrt{b+c} + \sqrt{c+a} \geq \sqrt{7(a+b+c) - 3}
\]

Solution:
Let $a + b + c = t$. Then since $1 = a^2 + b^2 + c^2 \leq (a + b + c)^2$ we get that $t \geq 1$. Note that $ab + bc + ca = \frac{t^2 - 1}{2}$. Straightforward calculations show that
\[
(\sqrt{a+b} + \sqrt{b+c} + \sqrt{c+a})^2 = 2t + 2(\sqrt{a^2 + \frac{t^2 - 1}{2}} + \sqrt{b^2 + \frac{t^2 - 1}{2}} + \sqrt{c^2 + \frac{t^2 - 1}{2}})(\dagger)
\]
Now let us show that
\[
\sqrt{a^2 + \frac{t^2 - 1}{2}} \geq a + \frac{t - 1}{2}
\]
(\dagger\dagger)
Indeed, by squaring of positive sides of (\dagger\dagger) we get an equivalent inequality
\[
a^2 + \frac{t^2 - 1}{2} \geq a^2 + a(t - 1) + \frac{(t - 1)^2}{4}
\]
which in turn is equivalent to $(t - 1)(t + 3 - 4a) \geq 0$. Since $t \geq 1 \geq a$ (\dagger\dagger) is proved. By inserting the inequality (\dagger\dagger) for $a, b$ and $c$ into (\dagger) we get
\[
(\sqrt{a+b} + \sqrt{b+c} + \sqrt{c+a})^2 \geq 7(a+b+c) - 3
\]
The equality holds at $t = 1$ (equivalently $(a, b, c) = (1, 0, 0), (0, 1, 0), (0, 0, 1)$). The proof is completed.