Problem: A sequence \( \{a_n\} \) is said to be good if \( a_1 = 1 \) and \( |a_{k+1}| = |a_k + 1| \). Let \( c_n = \min \sum_{i=1}^{n} |a_i| \), where the minimum is taken over all good sequences. Prove that the sequence \( \{c_n\} \) is unbounded from above.

Solution:
Note that \( a_1 = 1 \) and for all \( i \geq 1 \)

\[
a_{i+1}^2 = a_i^2 + 2a_i + 1 \tag{\dagger}
\]

The sum of (\dagger) over \( i = 1, \ldots, n \) yields:

\[
a_{n+1}^2 = 2 \sum_{i=1}^{n} a_i + n + 1 \text{ or } c_n = |a_{n+1}^2 - (n + 1)|.
\]

Now we note that since \((k + 1)^2 - k^2 = 2k + 1\), the distance from \( n + 1 \) to the nearest perfect square is unbounded when \( n \) increases. Done.