Problem 1.
(a) A graph has five vertices of degree 4 and two vertices of degree 2. How many edges does it have?
(b) A graph has degree sequence 5,5,4,4,3,3,3. How many edges does it have?

Problem 2. Let \(m\) and \(M\) denote the minimum and the maximum degrees of the vertices of a graph \(G\) with vertex set \(V\) and edge set \(E\). Show that
\[
m \leq \frac{2|E|}{|V|} \leq M.
\]

Problem 3. Suppose \(G_1\) and \(G_2\) are two graphs that have Euler circuits. Suppose \(G_1\) and \(G_2\) do not have vertices in common. Let \(v_1\) be a vertex in \(G_1\) and let \(v_2\) be a vertex in \(G_2\). Join \(v_1\) and \(v_2\) with a single edge. Does a resulting graph possess an Euler circuit? Explain.

Problem 4.
(a) Show that the Petersen graph has no Hamiltonian cycle but has a Hamiltonian path.
(b) Show that if any vertex (and the edges incident to it) is removed from the Petersen graph, then the resulting subgraph has a Hamiltonian cycle.

Problem 5. In a group of \(2n\) people, each person has at least \(n\) friends. Prove that the group can be seated in a circle, each person next to a friend.