Quiz 1

1. Write an IVP which is governed by a fourth-order, nonlinear ordinary differential equation.

\[ y^{(4)} - 4y'^3 + 2xy' - 8y = e^x \]
\[ y(1) = y'(1) = -2, \ y''(1) = 8, \ y'''(1) = 0. \]

2. Determine the values of “m” for which the differential equation

\[ y''' - 3y'' + 2y' = 0 \]

has solutions of the form \( y = e^{mx} \).

\[ (D^3 - 3D^2 + 2D)y = 0 \]

where \( D = \frac{d}{dt} \) and \( \frac{d}{dt}(e^{mx}) = me^{mx} \). Then

\[ me^{mx}(m^3 - 3m + 2) = 0 \Rightarrow m = 0 \text{ or } m = 1 \text{ or } m = 2. \]

3. Consider

\[ (x^2 + 4)y' = y - y^2, \ y(-1) = y_0, y_0 \in \mathbb{R}. \]

Find \( y_0 \) such that the resulting IVP has a unique solution, and find this unique solution.

\[ f(x, y) = \frac{y - y^2}{x^2 + 4} \text{ and } f_y(x, y) = \frac{1 - 2y}{x^2 + 4} \]

are continuous everywhere. Then the IVP has a unique solution in some interval about \( x_0 = -1 \).

If \( y_0 = 0 \), then \( y = 0 \) is the desired solution.

If \( y_0 = 1 \), then \( y = 1 \) is the only function which satisfies the d.e. and the initial condition.