Solution 1

(a) The graph of the function is given in Figure 1.

(b) The limits are \( \lim_{x \to 1^+} f(x) = 2 \) and \( \lim_{x \to 1^-} f(x) = 1 \).

(c) The limit \( \lim_{x \to 1} f(x) \) does not exist, since the right-hand and left-hand limits are not equal as \( x \to 1 \).

(d) The limits are

\[
\begin{align*}
\lim_{h \to 0^+} \frac{f(h)}{h} &= \lim_{h \to 0^+} \frac{h}{h} = 1 \\
\lim_{h \to 0^-} \frac{f(h)}{h} &= \lim_{h \to 0^-} \frac{-h}{h} = -1.
\end{align*}
\]

(e) The limit \( \lim_{h \to 0} \frac{f(h)}{h} \) does not exist, since the right-hand and left-hand limits are not equal as \( h \to 0 \).

Solution 2

(a) Since \( \lim_{u \to 0} \frac{\sin u}{u} = 1 \),

\[
\begin{align*}
\lim_{x \to 0} \frac{1 - \cos(x)}{x \sin x} &= \lim_{x \to 0} \frac{[1 - \cos(x)] [1 + \cos(x)]}{x \sin x [1 + \cos(x)]} \\
&= \lim_{x \to 0} \frac{1 - \cos^2(x)}{x \sin x} \\
&= \lim_{x \to 0} \frac{\sin^2(x)}{x \sin x} \\
&= \lim_{x \to 0} \frac{\sin(x) \sin(x) \sin x}{x \sin x} \frac{1}{x \sin x} \frac{1}{1 + \cos(x)} \\
&= 1 \cdot 1 \cdot 1 \cdot \frac{1}{1 + 1} = \frac{1}{2}
\end{align*}
\]

Thus

\[
\lim_{x \to 0} \frac{1 - \cos(x)}{x \sin x} = \frac{1}{2}
\]

(b) Since \( x - \sqrt{x} - 2 = (\sqrt{x} - 2)(\sqrt{x} + 1) \) and \( x - 4 = (\sqrt{x} - 2)(\sqrt{x} + 2) \), we have

\[
\begin{align*}
\lim_{x \to 4} \frac{x - 4}{x - \sqrt{x} - 2} &= \lim_{x \to 4} \frac{(\sqrt{x} - 2)(\sqrt{x} + 2)}{(\sqrt{x} - 2)(\sqrt{x} + 1)} \\
&= \lim_{x \to 4} \frac{\sqrt{x} + 2}{\sqrt{x} + 1} \\
&= \lim_{x \to 4} \frac{2 + 2}{2 + 1} = \frac{4}{3}
\end{align*}
\]

Thus

\[
\lim_{x \to 4} \frac{x - 4}{x - \sqrt{x} - 2} = \frac{4}{3}
\]
Solution 3

(a) The meaning of \( \lim_{x \to 9} \sqrt{x-5} = 2 \) is: for given \( \epsilon > 0 \) there exists a number \( \delta; > 0 \) such that for all \( x \) with 0 < \( |x - 9| < \delta \) we have \( |f(x) - 2| < \delta \).

(b) Using the definition in part (a)

**Step 1:** Solve the inequality \( |\sqrt{x-5} - 2| < \epsilon \) to find an interval about \( x_0 = 9 \) on which the inequality holds for all \( x \neq 9 \).

\[
|\sqrt{x-5} - 2| < \epsilon \\
-\epsilon < \sqrt{x-5} - 2 < \epsilon \\
2 - \epsilon < \sqrt{x-5} < 2 + \epsilon \\
(2 - \epsilon)^2 < x - 5 < (2 + \epsilon)^2 \\
9 - 4\epsilon + \epsilon^2 < x < 9 + 4\epsilon + \epsilon^2
\]

The inequality holds for all \( x \) in the open interval \((9 - 4\epsilon + \epsilon^2, 9 + 4\epsilon + \epsilon^2)\), so it holds for all \( x \neq 9 \) in this interval as well.

**Step 2:** Find a value of \( \delta > 0 \) that places the centered interval \(9 - \delta < x < 9 + \delta \) inside the interval \((9 - 4\epsilon + \epsilon^2, 9 + 4\epsilon + \epsilon^2)\). The distance from 9 to the nearer endpoint of \((9 - 4\epsilon + \epsilon^2, 9 + 4\epsilon + \epsilon^2)\) is \(4\epsilon - \epsilon^2\). If we take \( \delta = 4\epsilon - \epsilon^2 \) or any smaller positive number when \( 0 < \epsilon \leq 2 \), and we take \( \delta = (4)(2) - 2^2 = 4 \) when \( \epsilon > 2 \), then the inequality \( 0 < |x - 9| < \delta \) will place \( x \) between \( 9 - 4\epsilon + \epsilon^2 \) and \( 9 + 4\epsilon + \epsilon^2 \) to make \( |\sqrt{x-5} - 2| < \epsilon \). Hence,

\[
0 < |x - 9| < \delta \implies |\sqrt{x-5} - 2| < \epsilon.
\]

Solution 4

(a) \( \frac{dy}{dx} = f'(x) = -3(1 - 2x^2)^{-4}(-4x) = \frac{12x}{(1 - 2x^2)^4} \)

(b) Since \( \frac{d}{du} \sec u = \tan u \cdot \sec u \)

\[
\frac{dy}{dx} = f'(x) = \left\{2 \sec(\sec(x^2))\right\} \frac{d}{dx} \left[\sec(\sec(x^2))\right] \\
= \left\{2 \sec(\sec(x^2))\right\} \left\{\tan(\sec(x^2)) \sec(\sec(x^2))\right\} \frac{d}{dx} \left[\sec(x^2)\right] \\
= \left\{2 \sec(\sec(x^2))\right\} \left\{\tan(\sec(x^2)) \sec(\sec(x^2))\right\} \left\{\tan(x^2) \sec(x^2)\right\} \{2x\}
\]

(c) \( \frac{dy}{dx} = f'(x) = \sin(\sqrt{5x^2 + 1}) + x \cos(\sqrt{5x^2 + 1}) \left[\frac{10x}{2\sqrt{5x^2 + 1}}\right]. \)

Solution 5

We use Intermediate Value Theorem:

Suppose \( f(x) \) is continuous on an interval \( I \), and \( a, b \) are any two points of \( I \). Then, if \( y_0 \) is a number between \( f(a) \) and \( f(b) \), there exists a number \( c \) between \( a \) and \( b \) such that \( f(c) = y_0 \).

Consider \( f(x) = \pi \sin x - \pi + 2x \) over the interval \( 0 \leq x \leq \pi/2 \). The function is continuous everywhere and \( f(0) = -\pi < 0 < \pi = f\left(\frac{\pi}{2}\right) = \pi \). Hence, according to Intermediate Value Theorem, there exists a number \( c \) between 0 and \( \pi/2 \) such that \( f(c) = 0 \). The number \( c \) is the solution of the equation \( \pi \sin x = \pi - 2x \).
Solution 6
The equation of the tangent line to the curve \( y = x^2 - 4 \) at a point \((a, a^2 - 4)\) on the curve is
\[
y - (a^2 - 4) = 2a(x - a).
\]
Since the point \((3, 1)\) is on the line, we have
\[
1 - (a^2 - 4) = 2a(3 - a) \implies a^2 - 6a + 5 = 0 \implies (a - 1)(a - 5) = 0.
\]
Thus, the equations of two lines are
\[
y = 2x - 5 \quad \text{and} \quad y = 10x - 29.
\]

Solution 7
Differentiating the equation implicitly, we have
\[
3x^2 + 3y^2 \frac{dy}{dx} = 6y + 6x \frac{dy}{dx}.
\]
Setting \(x = 1\), and \(y = 2\), we obtain
\[
\frac{dy}{dx} = \frac{3}{2}.
\]
The equation of the tangent line at \((1, 2)\) is
\[
y = \frac{1}{2}(3x + 1) \quad \text{or} \quad 3x - 2y = -1,
\]
and the equation of the normal line at \((1, 2)\) is
\[
y = \frac{1}{3}(-2x + 8) \quad \text{or} \quad 2x + 3y = 8.
\]

Solution 8
A- True or False.
(a) True
(b) False
(c) False
(d) True
B- The correct answer is \(\text{(d)}\).