

5)(20 pts.) Show all your work.

Let $y^{(4)} - 5y'' + 4y = (1+x)e^x + 2e^{-x} + \sin 3x + x \cos 3x$ be the given 4-th order non-homogenous differential equation.

a) Find the general solution of the corresponding homogenous differential equation.

$$L(r) = r^4 - 5r^2 + 4 = (r^2 - 4)(r^2 - 1) = 0 \quad \text{and}$$

-2, 2, -1, 1 are the roots of this characteristic eqn.

Thus $y(x) = c_1 e^{-2x} + c_2 e^{2x} + c_3 e^{-x} + c_4 e^x$ is the general solution of the corresponding homog. dif. eqn.

b) Write down the form of the particular solution y_p , but **DO NOT calculate** the coefficients.

For the particular solution,

$(1+x)e^x$ suggests that $y_{p_1} = (A_1 + A_2 x)e^x$; A_1, A_2 are constants but e^x is also a solution of the homog. equation.

We should multiply y_{p_1} by x i.e. $y_{p_1} = (A_1 x + A_2 x^2)e^x$

$\sin 3x + x \cos 3x$ suggests that

$$y_{p_2} = (A_3 \cos 3x + A_4 \sin 3x) + x \cdot (A_5 \cos 3x + A_6 \sin 3x);$$

A_3, A_4, A_5, A_6 are constants.

$2e^{-x}$ suggests that $y_{p_3} = A_7 e^{-x}$; A_7 is constant but e^{-x} is also a solution of the homog. equation.

Thus we should multiply by x i.e. $y_{p_3} = A_7 x e^{-x}$.

Therefore

$$y_p = (A_1 x + A_2 x^2)e^x + (A_3 \cos 3x + A_4 \sin 3x) + x(A_5 \cos 3x + A_6 \sin 3x)$$

$$+ A_7 x e^{-x} = (A_1 x + A_2 x^2)e^x + \cos 3x(A_3 + xA_5) + \sin 3x(A_4 + xA_6) + A_7 x e^{-x}$$

is the particular solution.