

4) (20 pts.) Show all your work.

(*)

Given that $y_1(x) = \frac{1}{x}$ is a solution of $\underbrace{2x^2y'' + 3xy' - y = 0}_{x > 0}$,

find the general solution by using **reduction of order** and identify the second linearly independent solution. (**Other solutions will not be accepted**).

$$y(x) = v(x)y_1(x) = v(x)x^{-1}$$

$$y'(x) = v'x^{-1} - v x^{-2}$$

$$y''(x) = v''x^{-1} - 2v'x^{-2} + 2vx^{-3}$$

Put y, y' & y'' in (*)

$$2x^2(v''x^{-1} - 2v'x^{-2} + 2vx^{-3}) + 3x(v'x^{-1} - v x^{-2}) - vx^{-1} = 0$$

$$v \underbrace{(4x^{-1} - 3x^{-2} - x^{-1})}_0 + v'(-4 + 3) + v''(2x) = 0$$

Thus we get $2xv'' - v' = 0$

$$\text{Put } \omega = v'$$

$$2x\omega' - \omega = 0$$

$$\frac{\omega'}{\omega} = \frac{1}{2x} \text{ gives } \omega = x^{1/2} C_1$$

Using $\omega = v'$, we get

$$v' = x^{1/2} C_1 \Rightarrow v = \frac{2}{3} C_1 x^{3/2} + C_2$$

$$y(x) = v \cdot x^{-1} = \left(\frac{2}{3} C_1 x^{3/2} + C_2\right) x^{-1} = \frac{2}{3} C_1 x^{1/2} + C_2 x^{-1}$$

is the general solution & $y_2 = x^{1/2}$ is the second linearly independent solution.