

2) (20 pts.) Show all your work.

Let  $0 \neq C = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \in R^2$  and  $W_C = \{[x_1 \ x_2] \in R^2 \mid [x_1 \ x_2] \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = 0\}$  be given.

a) Show that  $W_C$  is a subspace of  $R^2$ .

$W_C \neq \emptyset$  because  $[0 \ 0] \in W_C$ .

Let  $[x_1 \ x_2]$  &  $[y_1 \ y_2]$  be in  $W_C$ . Then

$$\begin{aligned} ([x_1 \ x_2] + [y_1 \ y_2]) \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} &= [x_1 + y_1 \ x_2 + y_2] \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \\ &= c_1(x_1 + y_1) + c_2(x_2 + y_2) \\ &= (c_1 x_1 + c_2 x_2) + (c_1 y_1 + c_2 y_2) = 0 \end{aligned}$$

thus  $[x_1 \ x_2] + [y_1 \ y_2] \in W_C$

Let  $d$  be in  $R$ .

$$\begin{aligned} (d[x_1 \ x_2]) \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} &= [dx_1 \ dx_2] \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = dx_1 c_1 + dx_2 c_2 \\ &= d(c_1 x_1 + c_2 x_2) = 0 \end{aligned}$$

$\therefore d[x_1 \ x_2] \in W_C$  &  $W_C$  is a subspace of  $R^2$ .

b) Using the definition given part a), find  $W_C \cap W_D$  when  $C = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

and  $D = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ .

Let  $[x_1 \ x_2]$  be in  $W_C \cap W_D$ .

$$[x_1 \ x_2] \in W_C \Rightarrow [x_1 \ x_2] \begin{bmatrix} 1 \\ 2 \end{bmatrix} = x_1 + 2x_2 = 0$$

$$[x_1 \ x_2] \in W_D \Rightarrow [x_1 \ x_2] \begin{bmatrix} 1 \\ -1 \end{bmatrix} = x_1 - x_2 = 0$$

The solution set of this homogenous system has only the trivial solution because the coefficient matrix,  $A = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$  &  $\det A = -3 \neq 0$ , is invertible.

Therefore  $[0 \ 0] \in W_C \cap W_D$ .