

I) (20 pts.) Show all your work

Let A be a non-singular $n \times n$ matrix with $n > 1$.

a) Calculate $\det(\text{adj}A)$ in terms of $\det A$.

$$A^{-1} = \frac{1}{\det A} (\text{adj}' A)$$

$$\text{adj}' A = \det A \cdot A^{-1}$$

$$\det(\text{adj} A) = \det(\det A \cdot A^{-1})$$

$$= (\det A)^n \cdot \det(A^{-1})$$

$$= (\det A)^n \cdot (\det A)^{-1} = (\det A)^{n-1}$$

b) Show that $(\text{adj}A)^{-1} = (\det A^{-1})A = \text{adj}(A^{-1})$.

$$\text{adj}' A = \det A \cdot A^{-1}$$

$$(\text{adj} A)^{-1} = (\det A \cdot A^{-1})^{-1}$$

$$= (\det A)^{-1} \cdot A$$

$$= \det(A^{-1}) \cdot A = \text{adj}'(A^{-1})$$