

Math 225
2008-2009 Fall
Final Exam Questions

1) a) Let A be an $n \times n$ matrix. Show that A can be written as the sum of a symmetric and a skew symmetric matrices.

b) Suppose $\{u_1, u_2, u_3, u_4, u_5\}$ be a basis for R^5 . If c_1, c_2, c_3, c_4, c_5 are scalars with $c_3 \neq 0$, show that $\{u_1, u_2, c_1u_1 + c_2u_2 + c_3u_3 + c_4u_4 + c_5u_5, u_4, u_5\}$ is also a basis for R^5 .

2) Let $C[0,1]$ be the vector space of all continuous real valued functions with domain $[0,1]$. Let

$\langle f, g \rangle = \int_0^1 f(x)g(x)dx$ be the inner product in $C[0,1]$ where f and g are two functions in

$C[0,1]$. Answer the following questions for $f(x) = x$ and $g(x) = \cos x$.

a) Find $\langle f, g \rangle$ and $\|g\|$ where $\|\cdot\|$ denotes the **length** induced by this inner product. Show your work.

b) Determine the scalar c so that $f - cg$ is orthogonal to f . Show all your work.

3) Let $A = \begin{bmatrix} a & a & -3a \\ 0 & 2 & 9 \\ 0 & 1 & 2 \end{bmatrix}$ where a is a real number.

a) Find the characteristic polynomial of A . (Of course it may depend on a). **Show all your work.**

b) What must be a if A is **not** diagonalizable? **Explain your answer.**

4) Let $(D-1)^2(D^2+4)y = x^5e^x + 7\sin 2x$ be the given 4-th order nonhomogeneous differential equation with constant coefficients.

a) Find the general solution of the corresponding homogeneous differential equation.

b) Write down the form of the particular solution y_p **but DO NOT evaluate the coefficients.**

5) Use the method of variation of parameters to find a particular solution to the differential

equation $y'' - 3y' + 2y = \frac{1}{1+e^{-x}}$. **Show all your work.**