MATH 220
MIDTERM I
Spring 2017

NAME :

STUDENT I.D. :

THIS EXAMINATION PAPER CONTAINS 6 PAGES AND 4 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. PLEASE BRING ANY DISCREPANCY TO THE ATTENTION OF THE INVIGILATOR.

No books, notes or calculators of any kind are allowed. You must give detailed mathematical explanations for all your conclusions in order to receive full credit.

Duration of the examination is 110 minutes, starting at 15:40 and ending at 17:30.

GOOD LUCK !

1	2	3	4	Total
25	25	25	25	100

1. (25 points) Consider the following system of linear equations. Find for which values of c the system has a solution. For these values write solutions using parameters. You are expected to solve this problem using matrix methods, i.e., Gauss-Jordan elimination, by reducing the augmented matrix to reduced row echelon form. Do not use elimination method.

$$\begin{aligned}
2x_1 - 3x_2 + x_3 - x_4 + 2x_5 &= -2 \\
x_1 - 2x_2 + x_3 - x_4 + x_5 &= -2 \\
2x_1 \qquad - x_3 + 3x_4 + x_5 &= 3 \\
x_1 - x_2 + x_3 + x_4 \qquad &= c + 2
\end{aligned}$$

2. (25 points) Find the inverse A^{-1} of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & -2 \\ -1 & 0 & 1 & 1 \\ 2 & -1 & -1 & -1 \end{bmatrix}$$

using row operations. Solve the equation $A\mathbf{x} = \mathbf{b}$ for the matrix $\mathbf{b} = \begin{bmatrix} 2 \\ 5 \\ 1 \\ 3 \end{bmatrix}$ using A^{-1}.

3. a) (12 points) Let

$$A = \begin{bmatrix} 2 & -1 & 2 & -1 \\ 0 & -1 & 3 & 0 \\ -2 & 0 & -1 & 2 \\ 3 & 1 & 0 & -1 \end{bmatrix}.$$

Calculate $\det(A)$.

b) (13 points) Let B be a 5×5-matrix such that $\det(B) = 2$. Let C be the matrix obtained from B by multiplying its second column by -7. Let D be the matrix obtained from B by replacing its first row with its fourth row (Caution: replacing not interchanging !). Find the determinants $\det(B^{-1})$, $\det(C)$, $\det(D)$, $\det(BC)$, and $\det(B + C)$.

4. (a) For each of the statements below indicate whether the statement is always true or sometimes false. Justify your answer with a logical argument (for true statements) or give a counterexample (for false statements).

(i) (5 points) Let A and B be $n \times n$ matrices. If AB is invertible, then both A and B are invertible.

(ii) (5 points) If A is an $n \times n$ matrix such that $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions, then there is a nonzero matrix C such that $AC = 0$.

(iii) (5 points) If A and B are invertible $n \times n$ matrices, then $(A+B)^{-1} = A^{-1} + B^{-1}$.

4. (b) (10 points) The trace $\mathrm{Tr}(A)$ of an $n \times n$ matrix $A = (a_{ij})$ is defined as the sum of its diagonal entries, that is, $\mathrm{Tr}(A) = \sum_{i=1}^{n} a_{ii}$. Show that for every elementary matrix E, the equality $\mathrm{Tr}(AE) = \mathrm{Tr}(EA)$ holds.