MATH 220
MIDTERM 2
Fall 2018

NAME :

STUDENT I.D. :

SECTION :

THIS EXAMINATION PAPER CONTAINS 6 PAGES AND 5 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. PLEASE BRING ANY DISCREPANCY TO THE ATTENTION OF THE INVIGILATOR.

No books, notes or calculators of any kind are allowed. You must give detailed mathematical explanations for all your conclusions in order to receive full credit.

Duration of the examination is 120 minutes, starting at 18:00 and ending at 20:00.

GOOD LUCK !

1	2	3	4	5	Total
20	20	20	25	15	100

1. a) (10 pts) Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 0 \\ a \\ 2 \end{bmatrix}.$$

Find all the values for a so that the vector $\mathbf{v}$ is in the span of $\mathbf{v}_1$ and $\mathbf{v}_2$. Write the coordinates of $\mathbf{v}$ in with respect to $\{\mathbf{v}_1, \mathbf{v}_2\}$.

b) (10 pts) Let $\mathbf{u} = (1, -1, 2)^T$. The subspace U of $\mathbb{R}^3$ is defined by

$$U = \{\mathbf{v} \in \mathbb{R}^3 \,|\, \mathbf{v} \cdot \mathbf{u} = 0\}$$

Find a basis for U.

2. (20 pts) Consider the matrix

$$A = \begin{bmatrix} -2 & -4 & 1 & 0 & 4 \\ 0 & 0 & 0 & 3 & 3 \\ 1 & 2 & 2 & 1 & 4 \\ 3 & 6 & 0 & -2 & -5 \end{bmatrix}$$

Convert A to the reduced echelon form and answer the following questions.
a) Write down a basis for the column space of A. What is the rank of A?
b) Write down a basis for the subspace consisting of all the solutions to the equation $A\mathbf{x} = 0$.
c) Write down a basis for the subspace consisting of all possible vectors $\mathbf{b}$ such that $A^T\mathbf{x} = \mathbf{b}$ for some $\mathbf{x}$.

3. (10+10 pts) Let $S = \{u_1, u_2, u_3\}$ be a basis for $\mathbb{R}^3$, where

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}.$$

a) Applying the Gram-Schmidt process to the basis S find an orthogonal basis $T = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.
b) Find the change of basis matrix $P_{S \leftarrow T}$.

4. a) (12 points) Let A be an $m \times n$ matrix where $m \geq n$, and let B be an $n \times n$ matrix such that $AB = 0$. Suppose that the rank of B is r, then what can we say about the rank of A? Justify your answer with mathematical arguments.

b) (13 points) Let A be an $n \times n$ matrix. Suppose that the product defined on $\mathbb{R}^n$ with the formula $\langle \mathbf{v}, \mathbf{w} \rangle = \mathbf{v}^T A \mathbf{w}$ is an inner product. Show that the following two conditions hold: (i) A is invertible (ii) There is an invertible matrix P such that $A = P^T P$.

5. For each of the statements below indicate whether the statement is always true or sometimes false. Justify your answer with a logical argument.

(i) (5 pts) Consider the vector space M_{23} of 2×3 matrices. Let W be the subset of M_{23} formed by 2×3 matrices A whose nullity is equal to 1. Then W is a subspace of M_{23}.

(ii) (5 pts) For every nonzero square matrix A, we have $\mathrm{Adj}(A) \neq 0$.

(iii) (5 pts) If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a basis for a finite dimensional vector space V, then the set of vectors $\{\mathbf{u} - 2\mathbf{v} + 3\mathbf{w}, 2\mathbf{u} + \mathbf{v} - \mathbf{w}, \mathbf{u} - \mathbf{v} + \mathbf{w}\}$ is also a basis for V.