MATH 220
Fall 2019
Quiz 3

Solutions

Full Name/ Student ID:

1) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}.$$

6 **a)** Find the eigenvalues and eigenvectors of A.

4 **b)** Find a nonsingular matrix P such that $P^{-1}AP$ is diagonal. Calculate the inverse matrix P^{-1} and show that $P^{-1}AP$ is indeed diagonal.

a) $p(\lambda) = \det(\lambda I - A) = \begin{vmatrix} \lambda-1 & -2 & -3 \\ 0 & \lambda-1 & 0 \\ -2 & -1 & \lambda-2 \end{vmatrix} = (\lambda-1)\begin{vmatrix} \lambda-1 & -3 \\ -2 & \lambda-2 \end{vmatrix}$

$= (\lambda-1)(\lambda^2 - 3\lambda + 2 - 6) = (\lambda-1)(\lambda^2 - 3\lambda - 4) = (\lambda-1)(\lambda-4)(\lambda+1)$

$\boxed{\lambda_1 = 1 \mid \lambda_2 = 4 \mid \lambda_3 = -1}$

+6 Eigenvalues of A

$\boxed{\lambda_1 = 1}$

$$\begin{bmatrix} 0 & -2 & -3 \\ 0 & 0 & 0 \\ -2 & -1 & -\cancel{1} \end{bmatrix} \to \begin{bmatrix} 1 & 1/2 & 1/2 \\ 0 & 1 & 3/2 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -1/4 \\ 0 & 1 & 3/2 \\ 0 & 0 & 0 \end{bmatrix} \qquad \vec{v}_1 = \begin{bmatrix} 1/4 \\ -3/2 \\ 1 \end{bmatrix}$$

$\frac{1}{2} - \frac{3}{4}$

$\boxed{\lambda_2 = 4}$

$$\begin{bmatrix} 3 & -2 & -3 \\ 0 & 3 & 0 \\ -2 & -1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$\boxed{\lambda_3 = -1}$

$$\begin{bmatrix} -2 & -2 & -3 \\ 0 & -2 & 0 \\ -2 & -1 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3/2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} -3/2 \\ 0 \\ 1 \end{bmatrix}$$

$$+2 \quad P = \begin{bmatrix} 1/4 & 1 & -3/2 \\ -3/2 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{array}{l} 4\times \\ 2\times \\ \end{array}\left[\begin{array}{ccc|ccc} 1/4 & 1 & -3/2 & 1 & 0 & 0 \\ -3/2 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 4 & -6 & 4 & 0 & 0 \\ -3 & 0 & 0 & 0 & 2 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 3 & -7 & 4 & 0 & -1 \\ 0 & 3 & 3 & 0 & 2 & 3 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 2/3 & 1 \\ 0 & 3 & -7 & 4 & 0 & -1 \end{array}\right] \begin{array}{l} \\ \\ -3 \end{array}$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 2/3 & 1 \\ 0 & 0 & -10 & 4 & -2 & -4 \end{array}\right] \to \begin{array}{l} \\ -1 \\ \end{array}\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 2/3 & 1 \\ 0 & 0 & 1 & -2/5 & 1/5 & 2/5 \end{array}\right]$$

$$\to \begin{array}{l} \\ -1 \\ +1 \end{array}\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -2/3 & 0 \\ 0 & 1 & 1 & 0 & 2/3 & 1 \\ 0 & 0 & 1 & -2/5 & 1/5 & 2/5 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & 2/5 & 7/15 & 3/5 \\ 0 & 0 & 1 & -2/5 & 1/5 & 2/5 \end{array}\right]$$

$$\underset{(5)}{2/3} - \underset{(3)}{1/5} = \frac{10-3}{15}$$

+1 $$P^{-1} = \begin{bmatrix} 0 & -2/3 & 0 \\ 2/5 & 7/15 & 3/5 \\ -2/5 & 1/5 & 2/5 \end{bmatrix}.$$

+1 $$P^{-1}AP = \begin{bmatrix} 0 & -2/3 & 0 \\ 2/5 & 7/15 & 3/5 \\ -2/5 & 1/5 & 2/5 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1/4 & 1 & -3/2 \\ -3/2 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2/3 & 0 \\ 2/5 & 7/15 & 3/5 \\ -2/5 & 1/5 & 2/5 \end{bmatrix}\begin{bmatrix} 1/4 & 4 & 3/2 \\ -3/2 & 0 & 0 \\ 1 & 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix} = D = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}.$$

$1/4 - 3 + 3$

$-3/2 + 3$

$1/2 - 3/2 + 2$

-1

$-3 + 2$

$\frac{1}{10} + \frac{28}{15} + \frac{6}{10}$

$-7/10$

$-\frac{1}{10} + \frac{-3}{10} + \frac{4}{10}$

$-2/5 \quad -\frac{2}{5}$