MATH 220
Fall 2019
Quiz 2

Full Name/ Student ID: Solutions

1) Find a basis for the subspace W of $\mathbb{R}^3$ spanned by

$$\left\{ \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}3\\2\\1\end{bmatrix}, \begin{bmatrix}11\\10\\7\end{bmatrix}, \begin{bmatrix}7\\6\\4\end{bmatrix} \right\}.$$

What is the dimension of W?

$$\begin{matrix} v_1 \to \\ v_2 \to \\ v_3 \to \\ v_4 \to \end{matrix} \begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 1 \\ 11 & 10 & 7 \\ 7 & 6 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 2 \\ 0 & -4 & -5 \\ 0 & -12 & -15 \\ 0 & -8 & -10 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 5/4 \\ 0 & 1 & 5/4 \\ 0 & 1 & 5/4 \end{bmatrix} \to \begin{bmatrix} (1) & 2 & 2 \\ 0 & (1) & 5/4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \leftarrow w_1 \\ \leftarrow w_2 \\ \\ \\ \end{matrix}$$

$$S = \left\{ \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}0\\1\\5/4\end{bmatrix} \right\}$$ is a basis for W.

$\boxed{\dim W = 2}$

2. Find a basis for the solution space of the homogeneous system $A\underline{x} = \underline{0}$ where

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 2 & 1 & 2 \\ 2 & 4 & 3 & 3 & 3 \\ 0 & 0 & 1 & -1 & -1 \end{bmatrix}.$$

What is the dimension of the solution space?

$$\begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 2 & 1 & 2 \\ 2 & 4 & 3 & 3 & 1 \\ 0 & 0 & 1 & -1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}$$

$$\to \begin{bmatrix} (1) & 2 & 1 & 2 & 1 \\ 0 & 0 & (1) & -1 & 1 \\ 0 & 0 & 0 & 0 & (1) \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(column 2 ← s, column 4 ← t)

$\boxed{x_2 = s, \quad x_4 = t}$

$x_1 + 2x_2 + x_3 + 2x_4 + x_5 = 0$

$x_3 - x_4 + x_5 = 0$

$\boxed{x_5 = 0}$

$\Rightarrow x_3 = x_4 = t \qquad \boxed{x_3 = t}$

$\Rightarrow x_1 = -2s - t - 2t$

$\boxed{x_1 = -2s - 3t}$

$$\underline{x} = \begin{bmatrix} -2s-3t \\ s \\ t \\ t \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} s + \begin{bmatrix} -3 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} t$$

$$\text{Basis} = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}. \quad \boxed{\dim = 2}$$