MATH 220
Fall 2019
Quiz 1

Full Name/ Student ID: Solutions

1) Find the inverse, if it exists, of the following matrix.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-R1+R2\to R2} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\to R3]{-R2+R3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & -1 & 1 \end{array}\right]$$

$$\xrightarrow{-R3\to R3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right] \xrightarrow[-R3+R1\to R1]{-2R3+R2\to R2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right]$$

$$\xrightarrow{-R2+R1\to R1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right] \qquad A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 2 \\ -1 & 1 & -1 \end{bmatrix}$$

2. Let A and B be row equivalent $n \times n$ matrices. Prove that A is nonsingular if and only if B is nonsingular.

If A and B are row equivalent, then there exist elementary matrices, $E_1, \dots, E_k$ such that $B = E_k E_{k-1} \cdots E_1 \cdot A$.

Then the result follows from the following observation

Fact: If C and D are invertible, then $C \cdot D$ is invertible.
Let C^{-1} and D^{-1} are inverses of C and D. Then

$$(C\cdot D)\cdot(D^{-1}\cdot C^{-1}) = C(\underbrace{D\cdot D^{-1}}_{I})\cdot C^{-1} = \underbrace{C\cdot C^{-1}}_{I} = I$$

$$(D^{-1}\cdot C^{-1})\cdot(C\cdot D) = D^{-1}(\underbrace{C^{-1}\cdot C}_{I})D = D^{-1}D = I$$

$D^{-1}\cdot C^{-1}$ is inverse of $C\cdot D$.