MATH 220
HOMEWORK 4
Due December 6, 2019, Friday, submit at the beginning of the class.

Note: Each solution should start at the top of a page (you can use both sides of the paper) and each sheet should have your name on it. Note that this homework has 5 questions in 2 pages.

1. Let P_2 denote the vector space of polynomials with degree ≤ 2. Let $S = \{f_1, f_2, f_3, f_4, f_5\}$ be a set of vectors in P_2, where $f_1(x) = x^2+3x+4$, $f_2(x) = x + 1$, $f_3(x) = x^2 + x + 2$, $f_4(x) = x^2 - x - 2$, $f_5(x) = 3x - 5$.

   a) Show that S spans P_2.

   b) Find a <u>subset</u> of S which is a basis for P_2.

   c) Find the coordinate vector $[f(x)]_S$ of the vector $f(x) = x^2 + 3x + 1$ with respect to the basis S that you found in part (b).

2. Consider the following matrix

$$A = \begin{bmatrix} 1 & 3 & -1 & -1 & 0 \\ -1 & -2 & 0 & 2 & 2 \\ 0 & -3 & 2 & 0 & 1 \\ 1 & 1 & 0 & 0 & 3 \end{bmatrix}$$

   a) Find a basis for the row space of A. What is the row rank of A?

   b) Find a basis for the column space of A. What is the column rank of A?

   c) Find a basis for the null space of A. What is the nullity of A?

3. Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ and $T = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ be ordered basis for R^3, where

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}.$$

   Suppose that the transition matrix from T to S is

$$P_{S \leftarrow T} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ -2 & 1 & -1 \end{bmatrix}.$$

Determine T (i.e. find the vectors $\mathbf{w}_1$, $\mathbf{w}_2$ and $\mathbf{w}_3$).

4. Let $\mathbb{R}_3$ have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ into an orthonormal basis, where
$$\mathbf{u}_1 = (1, -1, 1), \ \mathbf{u}_2 = (0, 1, -1), \ \mathbf{u}_3 = (1, -2, 1).$$
Check you answer using Matlab (write down which commands and algorithms that you used).

5. Decide whether the following sentences are true or sometimes false. If it is true, then give a logical argument to justify it. If it is false, then give an example that illustrates why it is false.

   a) Let V be a vector space and S_1, S_2 be two sets of vectors in V such that $S_1 \subseteq S_2$. If S_1 spans V, then S_2 spans V.

   b) Let A be a $n \times n$ matrix whose determinant is nonzero. If $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is a basis for $\mathbb{R}^n$, then $\{A\mathbf{v}_1, \ldots, A\mathbf{v}_n\}$ is a basis for $\mathbb{R}^n$.

   c) Let A be an $m \times n$-matrix. If there is an $n \times n$ matrix $B \neq 0$ such that $AB = 0$, then the rank of A is strictly smaller than m.