MATH 220
HOMEWORK 3
Due November 15, 2019, Friday, submit at the beginning of the class.

Note: Each solution should start at the top of a page (you can use both sides of the paper) and each sheet should have your name on it. The homework has 4 questions in 2 pages.

1. Determine which of the following sets are vector spaces under the given operations. For those that are not, list some of the axioms that fail to hold.

   a) The set of all triples of real numbers (x, y, z) with the operations

   $$(x, y, z) + (x', y', z') = (x + x', y + y', z + z') \quad \text{and} \quad \lambda(x, y, z) = (\lambda x, y, z)$$

   b) The set of all pairs of real numbers of the form $(1, y)$ with the operations

   $$(1, y) + (1 + y') = (1, y + y') \quad \text{and} \quad \lambda(1, y) = (1, \lambda y)$$

   c) The set of all real valued functions f defined everywhere on real line and such that $f(1) = 0$ with the standard operations

   $$(f + g)(x) = f(x) + g(x) \quad \text{and} \quad (\lambda f)(x) = \lambda f(x).$$

2. Let M_{22} denote the vector space of all 2×2 matrices with real number entries. Determine which of the following are subspaces of M_{22}.

   a) All 2×2 matrices of the form

   $$\begin{bmatrix} a & a+b \\ a+b & b \end{bmatrix}$$

   b) All 2×2 matrices

   $$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

   where $a^2 + b = d$.

   c) All 2×2 matrices A such that $\det A = 0$.

3. a) Let $\mathbf{v}_1 = (2, 1, 0, 3)$, $\mathbf{v}_2 = (3, -1, 5, 2)$, and $\mathbf{v}_3 = (-1, 0, 2, 1)$. Decide whether or not $\mathbf{v} = (2, 3, -7, 3)$ is in the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

   b) Let $\mathbb{R}_3$ denote the vector space of 1×3 row matrices. Decide whether or not the set of vectors

$$\{[3\ 1\ 1],\ [2\ -1\ 5],\ [4\ 0\ -3]\}$$

   in $\mathbb{R}_3$ is linearly independent?

4. Find the coordinate vector of $\mathbf{v}$ relative to the basis $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

   a) $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$.

   b) $\mathbf{v} = \begin{bmatrix} 5 \\ -12 \\ 3 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -4 \\ 5 \\ 6 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 7 \\ -8 \\ 9 \end{bmatrix}$.