Math 220 - Homework 2 - Solutions

1

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 3 & -4 & 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R1+R3\to R3]{-3R1+R2\to R2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & -1 & -5 & -3 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 & 1 \end{array}\right] \xrightarrow[\frac{1}{2}R3\to R3]{-R2\to R2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 5 & 3 & -1 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & 0 & \frac{1}{2} \end{array}\right]$$

$+1$

$$\xrightarrow{R2+R1\to R1} \left[\begin{array}{ccc|ccc} 1 & 0 & 7 & 4 & -1 & 0 \\ 0 & 1 & 5 & 3 & -1 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & 0 & \frac{1}{2} \end{array}\right] \xrightarrow[-7R3+R1\to R1]{-5R3+R2\to R2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -1 & -\frac{7}{2} \\ 0 & 1 & 0 & \frac{1}{2} & -1 & -\frac{5}{2} \\ 0 & 0 & 1 & \frac{1}{2} & 0 & \frac{1}{2} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & -1 & -\frac{7}{2} \\ \frac{1}{2} & -1 & -\frac{5}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}. \qquad \underline{x} = A^{-1}\cdot \underline{b} = \begin{bmatrix} \frac{1}{2} & -1 & -\frac{7}{2} \\ \frac{1}{2} & -1 & -\frac{5}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ 2 \end{bmatrix}$$

<u>Check</u>

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & -4 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -5 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -5+3+4 \\ -15+12+2 \\ 5-3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \checkmark$$

2

i) A = [1 -1 2 ; 3 0 1 ; -1 2 1]

ii) A(2,:) = (-3)*A(1,:) + A(2,:)

iii) format rat

iv) B = inv(A)

v) rref(B).

$$B = \begin{bmatrix} 13/14 & 5/14 & -1/14 \\ 5/14 & 3/14 & 5/14 \\ 3/14 & -1/14 & 3/14 \end{bmatrix}.$$

(3) $\begin{vmatrix} 1 & -2 & 3 \\ -4 & 1 & -1 \\ 1 & 2 & 2 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 3 \\ 0 & -7 & 11 \\ 0 & 4 & -1 \end{vmatrix} = \begin{vmatrix} -7 & 11 \\ 4 & -1 \end{vmatrix} =$

$4R1+R2 \to R2$
$-R1+R3 \to R3$

$= 7 - 44 = -37.$

$$\begin{vmatrix} 1 & -1 & 0 & 1 \\ 0 & 1/2 & 0 & 0 \\ -2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1/2 & 0 \\ -2 & 0 & 3 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 0 \\ -2 & 3 \end{vmatrix} = 3/2$$

(4) (i) FALSE. If we take $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$, then

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is invertible.

(ii) If A is a square matrix which has a column of zeros

TRUE then for some j, $a_{ij} = 0$ for all i.

This gives $(A^2)_{ij} = \sum_{k=0}^{n} a_{ik} \underset{0}{\underset{\parallel}{a_{kj}}} = 0$ for all i.

So, A^2 has all zeros at the j^{th} column.

(iii) TRUE. If A is not invertible, then there is elementary matrices $E_1, -, E_k$ such that

$$E_k E_{k-1} \cdots E_2 E_1 A = rref(A)$$

has a row of zeros. If we take $B' = \begin{bmatrix} 0 & 0 & \cdots & 1 & * \\ 0 & \cdots & & 1 & * \\ 0 & \cdots & & 1 & * \end{bmatrix}_{n \times n}$

(only nonzero at the last column, then

$0 = B' \cdot rref(A) = \underbrace{B' \cdot E_k \cdots E_2 E_1}_{B} \cdot A$. Take $B = B' E_k \cdots E_1$.