MATH 220
HOMEWORK 2
Due October 25, 2019, Friday, submit at the beginning of the class

Note: Each solution should start at the top of a page (you can use both sides of the paper) and each sheet should have your name on it. The homework has 4 questions.

1. Consider the linear system $A\underline{x} = \underline{b}$ where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -4 & 1 \\ -1 & 1 & 0 \end{bmatrix} \quad \text{and} \quad \underline{b} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

   Find A^{-1} if it exists. Decide if the system has a unique solution. If so, write the unique solution.

2. Let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}.$$

   Using Matlab perform the following operations in the given order.
   (i) Define a matrix A given as above.
   (ii) Perform the row operation $-3R1 + R2 \to R2$ on A.
   (iii) The new matrix is still called A. Switch Matlab to rational coefficients mode.
   (iv) Define B as the inverse matrix of A.
   (v) Find the reduced row echelon form of B.
   Write all the Matlab commands you used and write the matrix B as answers to your homework (or take print out if you can and attach to the homework). You can use Chapter 9 of our book to find the necessary Matlab commands.

3. Calculate the determinants of the following matrices.

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 1 & -1 \\ 1 & 2 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 1/2 & 0 & 0 \\ -2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. For each of the statements below indicate whether the statement is always true or sometimes false. Justify your answer with a logical argument (for true statements) or give a counterexample (for false statements).

   (i) If A is a 2×3 matrix and B is a 3×2 matrix, then AB is not invertible.

   (ii) If A is a square matrix which has a column of zeros, then A^2 has a column of zeros.

   (iii) If A is an $n \times n$ square matrix which is not invertible, then there exists a nonzero $n \times n$ matrix B such that $BA = 0$.