Math 220 – Homework 1 Solutions

(1)

$$\left[\begin{array}{rrr|r} 2 & 4 & -3 & 3 \\ 1 & 3 & -5 & -2 \\ 3 & 6 & -5 & 4 \end{array}\right] \xrightarrow{R1 \leftrightarrow R2} \begin{array}{r} \\ -2 \\ -3 \end{array}\left[\begin{array}{rrr|r} 1 & 3 & -5 & -2 \\ 2 & 4 & -3 & 3 \\ 3 & 6 & -5 & 4 \end{array}\right] \xrightarrow[-3R1+R3 \to R3]{-2R1+R2 \to R2} \begin{array}{r} \\ \\ -1 \end{array}\left[\begin{array}{rrr|r} 1 & 3 & -5 & -2 \\ 0 & -2 & 7 & 7 \\ 0 & -3 & 10 & 10 \end{array}\right]$$

$$\xrightarrow[\to R2]{-R3+R2} \begin{array}{r} \\ \\ +3 \end{array}\left[\begin{array}{rrr|r} 1 & 3 & -5 & -2 \\ 0 & 1 & -3 & -3 \\ 0 & -3 & 10 & 10 \end{array}\right] \xrightarrow{3R2+R3 \to R3} \left[\begin{array}{rrr|r} 1 & 3 & -5 & -2 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$x + 3y - 5z = -2$$
$$y - 3z = -3$$
$$\boxed{z = 1}$$

$$y = -3 + 3z = 0 \qquad \boxed{y = 0}$$

$$x = -2 - 3y + 5z = -2 + 5 = 3 \qquad \boxed{x = 3}$$

Solution

$$\boxed{x = 3 \;\big|\; y = 0 \;\big|\; z = 1}$$

---

(2) a)

$$BC = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \\ -1 & 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 9 & 14 \\ 17 & 25 & 27 \\ 11 & 3 & 6 \end{bmatrix}$$

$$\neq$$

$$CB = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \\ -1 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 14 & 9 & 35 \\ -2 & -4 & 2 \\ 1 & 14 & 32 \end{bmatrix}$$

b) $A^T(B+C) =$

$$= \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 9 & 4 \\ 2 & 1 & 6 \\ 2 & 2 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 28 & 14 \\ 6 & 4 & 20 \end{bmatrix}$$

---

(3) Plugging in $(x,y) = (-1,1), (0,-4), (2,-2)$ gives a system of equations for a, b, c.

$$\left.\begin{array}{r} a - b + c = 1 \\ \boxed{c = -4} \\ 4a + 2b + c = -2 \end{array}\right]$$

$$2/\ a - b = 5$$
$$+\ \ 4a + 2b = 2$$
$$6a = 12$$
$$\boxed{a = 2}$$

$$b = a - 5$$
$$\boxed{b = -3}$$

Answer:

$$\boxed{y = 2x^2 - 3x - 4}$$

(4)

$$\begin{array}{r}\\ \\ \\ -1\end{array}\left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 2 \\ 2 & 6 & -5 & -2 & 4 & -3 & c+1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 8 \end{array}\right] \to \begin{array}{r}\\ -2\\ \\ -1\end{array}\left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 2 \\ 2 & 6 & -5 & -2 & 4 & -3 & c+1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 0 & 0 & 5 & 10 & 0 & 21 & 7-c \end{array}\right]$$

$$\to \begin{array}{r}\\ \\ \frac{1}{5}\times\\ \frac{1}{6}\times\end{array}\left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 2 \\ 0 & 0 & -1 & -2 & 0 & -3 & c-3 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 0 & 0 & 0 & 0 & 0 & 6 & 2-c \end{array}\right] \to \begin{array}{r}\\ \\ +1\\ \\ \end{array}\left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 2 \\ 0 & 0 & -1 & -2 & 0 & -3 & c-3 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{2-c}{6} \end{array}\right]$$

$$\to \left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & c-2 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{2-c}{6} \end{array}\right] \to \begin{array}{r}+2\\ \\ -3 \text{ First}\\ \\ \end{array}\left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$c-2=0$

$\boxed{c=2}$ Condition for consistency.

$$\begin{array}{ccccccc} & s & & t & u & & \end{array}$$

$$\to \left[\begin{array}{cccccc|c} 1 & 3 & 0 & 4 & 2 & 0 & 4 \\ 0 & 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Reduced Row Echelon Form.

$x_1 + 3x_2 + 4x_4 + 2x_5 = 4$

$x_3 + 2x_4 = 1$

$\boxed{x_6 = 0}$

$$\boxed{\begin{array}{ll} x_2 = s & x_3 = 1-2t \\ x_4 = t & x_1 = 4-3s-4t-2u \\ x_5 = u & x_6 = 0 \end{array}}$$

Check: Put $c=2$.

$x_1=4$, $x_3=1$ is a particular solution for the system.

$x_1=-3$ $x_2=1$ solution for $A\underline{x}=\underline{0}$

$x_1=-4$, $x_3=-2$, $x_4=1$ sol. for $A\underline{x}=\underline{0}$

$x_1=-2$, $x_5=1$ is " " $A\underline{x}=\underline{0}$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} s + \begin{bmatrix} -4 \\ 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} t + \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u$$