Bilkent University Physics Department

Phys 552 (Spring 2009) Homework 2 Due March 6 2009

Question: Consider a particle in a one-dimensional harmonic oscillator potential. The probability distribution of the particle's position and velocity as a function of time is given by $P(x, v, t)$ where x and v are the random variables corresponding to the coordinate and velocity for the one-dimensional motion. The time dependence of these variables is given by the Langevin equation

$$\begin{aligned} \frac{dx}{dt} &= v \\ \frac{dv}{dt} &= \frac{1}{m}(-kx - \gamma v + n(t)) \end{aligned}$$

where k is the spring constant of the harmonic oscillator potential, γ is the viscous friction constant, and $n(t)$ is a random fluctuation (white noise) force with zero mean and variance σ.

(a) Construct the Fokker-Planck equation for the time dependence of $P(x, v, t)$.

(b) Show that the equilibrium distribution is of the form $P_{eq}(x, t) \propto \exp(-ax^2 - bv^2)$ and determine the constants a and b in terms of the given quantities.

(c) Show that the equilibrium distribution is consistent with the form $P_{eq}(x, t) \propto \exp(-E/k_B T)$, where E is the total energy of the particle. What relation must exist between the constants related to fluctuation and dissipation for this form to be valid?