1.(a) Note that the Hamiltonian is of the structure

$$H = E_o \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + E_o \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The first and third rows (coulumns) of the first matrix is of the form of the Pauli matrix σ_x so the eigenvalues and eigenvectors are:

$$E_1 = E_o \quad \psi_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \frac{1}{\sqrt{2}} \quad \Bigg| \quad E_2 = -E_o \quad \psi_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \frac{1}{\sqrt{2}} \quad \Bigg| \quad E_3 = E_o \quad \psi_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

(b) For the non-degenerate eigenstate, we have $E_2^{(1)} = \psi_2^{*T} H^{(1)} \psi_2 = -\epsilon$

For the degenerate states, we will have to form the matrix with elements $H^{(1)}_{\alpha\beta}$ where $\psi_\alpha = \psi_1$ and $\psi_\beta = \psi_3$:

$$H^{(1)}_{\alpha\beta} \to \epsilon \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \text{ which results in eigenvalues } \Delta E^{(1)} = \epsilon,\ 3\epsilon$$

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2. (a) $\psi_o(x) = A\exp(-x^2/x_o^2) \quad \Longrightarrow \quad |A|^2 \int_{-\infty}^{\infty} \exp(-2x^2/x_o^2)dx = 1 \quad \Longrightarrow \quad |A|^2 = \sqrt{2/\pi}/x_o$

$\psi_1(x) = a_+\psi_o(x) = (x/x_o - iP/p_o)A\exp(-x^2/x_o^2) = A[x/x_o + (\hbar/p_o)2x/x_o^2]\exp(-x^2/x_o^2) = (2x/x_o)A\exp(-x^2/x_o^2)$

(b) $\langle 1|V_o \exp(-t^2/\tau^2)\cos(kx - \Omega t)|0\rangle \exp(i\omega t) \underset{RWA}{\approx} \langle 1|V_o \exp(-t^2/\tau^2)\exp(ikx - i\Omega t)|0\rangle \exp(i\omega t)$

$\quad = V_o \langle 1|\exp(ikx)|0\rangle \exp(-t^2/\tau^2)\exp(i\omega t - i\Omega t)$

$\langle 1|\exp(ikx)|0\rangle = |A|^2 \int_{-\infty}^{\infty} (2x/x_o)\exp(-2x^2/x_o^2 + ikx)dx$

$\quad = |A|^2 \int_{-\infty}^{\infty} (2x/x_o)\exp[-(2/x_o^2)(x - ikx_o^2/4)^2 - k^2x_o^2/8]dx \qquad$ set $u = x - ikx_o^2/4$

$\quad = |A|^2 \exp(-k^2x_o^2/8) \int_{-\infty}^{\infty} [2(u + ikx_o^2/4)/x_o]\exp[-(2/x_o^2)u^2]du = (ikx_o/2)\exp(-k^2x_o^2/8)$

(c) $c_b^{(1)}(\infty) = (V_o/i\hbar)\langle 1|\exp(ikx)|0\rangle \int_{-\infty}^{\infty} \exp(-t^2/\tau^2)\exp(i\omega t - i\Omega t)dt$

$\quad = (V_o/i\hbar)\langle 1|\exp(ikx)|0\rangle \int_{-\infty}^{\infty} \exp[(-1/\tau^2)(t - i(\omega - \Omega)\tau^2/2)^2 - (\omega - \Omega)^2\tau^2/4]\,dt$

$\quad = (V_o/i\hbar)\langle 1|\exp(ikx)|0\rangle\, \tau\sqrt{\pi}\exp[-(\omega - \Omega)^2\tau^2/4]$

$P_b^{(1)} = |c_b^{(1)}(\infty)|^2 = (V_o\tau/\hbar)^2\pi(k^2x_o^2/4)\ \exp(-k^2x_o^2/4)\ \exp[-(\omega - \Omega)^2\tau^2/2]$

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3. (a) We have $\frac{\partial}{\partial\hbar}H = 2K/\hbar \quad \Longrightarrow \quad < K >= \frac{\hbar}{2}\frac{\partial}{\partial\hbar}E_n = \hbar\omega_o\left(n + \frac{1}{2}\right)/2 - \left[\hbar\omega_o(n + \frac{1}{2})\right]^2/(4V_o)$

(b) For $\frac{\partial}{\partial V_o}H = V/V_o$ so that $V_o\frac{\partial}{\partial V_o}E_n =< V >$. We have to be careful because ω_o contains a $\sqrt{V_o}$ factor. The second term in E_n has no V_o dependence and we have $< V >= \hbar\omega_o\left(n + \frac{1}{2}\right)/2$.

(c) We indeed have $< K > + < V > = E_n$.