

A spin-zero boson with mass  $m$  and charge  $q$  is in the first excited state of a three dimensional harmonic oscillator with a classical frequency  $\omega_c$ .

- (a) Separate the time-independent Schrödinger equation in the  $x$ ,  $y$ , and  $z$  coordinates, and identify the energy levels. Note that the first excited state is three fold degenerate (the  $|001\rangle$ ,  $|010\rangle$ , and  $|100\rangle$  states).
- (b) A magnetic field  $B_z$  in the  $z$  direction is now applied to the system so that the energy has an additional term  $H_1 = -qB_zL_z/(2m)$ . Treat this interaction as a perturbation, and use degenerate perturbation theory to find corrections to the original energy level.
- (c) How do you interpret the states that arise when the degeneracy is lifted?

Hint: Remember that  $L_z = xp_y - yp_x$ . Write down  $x$ ,  $p_x$ ,  $y$ , and  $p_y$  in terms of the raising and lowering operators  $a_{x\pm}$  and  $a_{y\pm}$ . (Remember that you will need separate raising and lowering operators for the  $x$  and  $y$  directions.) This will result in simpler algebra when constructing the matrix elements of  $H_1$  with respect to the degenerate states.

**The following problem is assigned to those students who did not attend class on Thursday, March 15th. Grade from this part of the homework will enter the geometric average of the grades of these students separately, so it must be completed in order to obtain a finite average. Other students may turn it in for extra credit.**

Repeat the above problem if the particle is in the first excited state in a cubic box (with infinitely high potential outside) of size  $a$ . Take the  $z$  direction parallel to one of the edges of the cube.