Second Exam Solutions Spring 2023

$$\begin{aligned} \overline{1.(a)} \quad \vec{E}_{1} &= \hat{y}E_{o}\exp[i(\hat{z}+\hat{x})k\cdot\vec{r}/\sqrt{2}-\omega t)] & \vec{E}_{2} &= \hat{y}E_{o}\exp[i(\hat{z}-\hat{x})k\cdot\vec{r}/\sqrt{2}-\omega t)] \\ \vec{B}_{1} &= \vec{k}_{1}\times\vec{E}_{1}/\omega = (-\hat{x}+\hat{z})(E_{o}/\sqrt{2}c)\exp[i(\hat{z}+\hat{x})k\cdot\vec{r}/\sqrt{2}-\omega t)] \\ \vec{B}_{2} &= \vec{k}_{2}\times\vec{E}_{2}/\omega = (-\hat{x}-\hat{z})(E_{o}/\sqrt{2}c)\exp[i(\hat{z}-\hat{x})k\cdot\vec{r}/\sqrt{2}-\omega t)] \\ (b) \quad \overline{\vec{S}} &= \operatorname{Re} \ (\vec{E}_{1}+\vec{E}_{2})\times(\vec{B}_{1}+\vec{B}_{2})^{*}/2\mu_{o} \\ &= \operatorname{Re} \ (2E_{o})\hat{y}\cos(kx/\sqrt{2})\exp[i(kz/\sqrt{2}-\omega t)] \\ &\times (2E_{o}/\sqrt{2}c)\{-\hat{x}\cos(kx/\sqrt{2})-i\hat{z}\sin(kx/\sqrt{2})\}\exp[-i(kz/\sqrt{2}-\omega t)]/2\mu_{o} \\ &= \hat{z}(\sqrt{2}E_{o}^{2}/Z_{o})\cos^{2}(kx/\sqrt{2}) \text{ with } Z_{o} &= \sqrt{\mu_{o}/\epsilon_{o}} \end{aligned}$$

2.(a) $\vec{E_o} = V/d$ (b) $B_o = k/\omega \hat{z} \times \vec{E_o} = -\hat{x}V/dc$ (c) B.C. at conductor surface: $K = \Delta H = V/(dc\mu_o)$ (In the \hat{z} direction for the positive conductor and the $-\hat{z}$ direction for the negative conductor.) $I = K w = wV/(dc\mu_o)$

(d) $Z_c = V/I = (d/w) c\mu_o = (d/w)Z_o$ with $Z_o = \sqrt{\mu_o/\epsilon_o}$.

3.(a) Flux in the loop: $\Phi = B_o \cos(\omega t)\pi R^2$ in the +z direction. The EMF produced will be $E = -d\Phi/dt = B_o\omega\sin(\omega t)$. It will be in a direction to oppose the change in the flux with the current it is producing. At $\omega t = \pi/6$ we have $\cos(\pi/6)$ decreasing, so that $\sin(\pi/6)$ positive, indicating generating flux in +z-direction. This implies current will be in the $+\hat{\phi}$ direction. (b) $-\partial \vec{B}/\partial t = \nabla \times \vec{E} = \nabla \times 2\hat{x}\sin(\omega t + \pi y) = -2\hat{z} \ \partial/\partial y \ \sin(\omega t + \pi y) = -2\pi\hat{z}\cos(\omega t + \pi y)$ $\vec{B} = 2\pi\hat{z}\sin(\omega t + \pi y)/\omega$ $\vec{H} = 2\pi\hat{z}\sin(\omega t + \pi y)/(\omega\mu_0)$