

1.(a) Charge on the conductor will distribute itself so that there will be zero field inside the conductor.

This implies the inner surface of the conductor will have charge $-\lambda$ per unit length, and the outer surface will have the remaining $\lambda + \gamma$ per unit length. then we will have $\sigma_{in} = \lambda/(2\pi a)$ and $\sigma_{out} = (\lambda + \gamma)/(2\pi b)$.

(b) Applying Gauss Law, $E_{in} = \lambda/(2\pi\epsilon_0 r)$ for $r < a$, field is zero inside the conductor for $a < r < b$, and finally, $E_{out} = (\lambda + \gamma)/(2\pi\epsilon_0 r)$ for $r > b$.

(c) We will have lower potential than $V(b) = 0$ for $r > b$ so that

$$V(r) = V(b) - \int_b^r dr (\lambda + \gamma)/(2\pi\epsilon_0 r) = -(\lambda + \gamma)/(2\pi\epsilon_0) \ln(r/b).$$

All points on the conductor will be at potential $V(b) = 0$ for $a < r < b$.

For points inside the conductor, we need to reference the potential at $V(a) = 0$ so that

$$V(r) = V(a) + \int_r^a dr \lambda/(2\pi\epsilon_0 r) = \lambda/(2\pi\epsilon_0) \ln(a/r) \text{ for } r < a.$$

2. (a) The relative position of Q with respect to the dipole is $-R\hat{x}$. The electric field at the position of Q will then be $(3p\hat{x} \cdot (-\hat{x}))(-\hat{x}) - (p\hat{x})/(4\pi\epsilon_0 R^3) = 2p\hat{x}/(4\pi\epsilon_0 R^3)$.

The force on the charge will then be $\vec{F}_Q = pQ\hat{x}/(2\pi\epsilon_0 R^3)$.

(b) The force on the dipole will be $\vec{F}_p = (\vec{p} \cdot \nabla) \vec{E} = p/(4\pi\epsilon_0) \frac{\partial}{\partial x} Q(x\hat{x} + y\hat{y} + z\hat{z})/(x^2 + y^2 + z^2)^{3/2} \big|_{(R,0,0)}$.

So that $\vec{F}_p = pQ/(4\pi\epsilon_0) [\hat{x}/r^3 - (3/2)\vec{r} 2x/r^5]_{(R,0,0)} = -pQ\hat{x}/(2\pi\epsilon_0 R^3) = -\vec{F}_Q$ as expected.

3. (a) The total charge is $\int_{-a}^{+a} \alpha z dz = 0$.

(b) $V(z) = 1/(4\pi\epsilon_0) \int_{-a}^{+a} \alpha z' dz'/(z - z')$.

Use the transformation $u = z - z'$ so that $dz' = -du$ and $z' = z - u$.

We then have $V(z) = \alpha/(4\pi\epsilon_0) \int_{-a}^{+a} (z - u) (-du)/u = \alpha/(4\pi\epsilon_0) \int_{-a}^{+a} (1 - z/u) du = \alpha/(4\pi\epsilon_0) [u - z \ln u]_{-a}^{+a}$.

Going back to the z' variable, $V(z) = \alpha/(4\pi\epsilon_0) [(z - z') - z \ln(z - z')]_{-a}^{+a} = \alpha/(4\pi\epsilon_0) [-2a - z \ln \frac{z-a}{z+a}]$

(c) Using the expansion $\ln(1 + \epsilon) \sim \epsilon - \frac{1}{2}\epsilon^2 + \frac{1}{3}\epsilon^3 + \dots$, we get

$$V(z) = \alpha/(4\pi\epsilon_0) [-2a + z \ln(1 + a/z) - z \ln(1 - a/z)]$$

$$\sim \alpha/(4\pi\epsilon_0) [-2a + z(a/z - \frac{1}{2}a^2/z^2 + \frac{1}{3}a^3/z^3 + \dots + a/z + \frac{1}{2}a^2/z^2 + \frac{1}{3}a^3/z^3 + \dots)] \sim \alpha a^3/(6\pi\epsilon_0 z^2).$$

Note the $1/z^2$ behavior.