1.(a) Potential due to the three charges: $V(z) = q[1/(z-a) - 2 + 1/(z+a)]/(4\pi\epsilon_o)$

(b) Expansion in powers of $1/z$: $V(z) = q[1/(1-a/z) - 2 + 1/(1+a/z)]/(4z\pi\epsilon_o)$

$$= q[1 + (a/z) + (a/z)^2 + \cdots - 2 + 1 - (a/z) + (a/z)^2 + \cdots]/(4z\pi\epsilon_o)$$

$$= q[(a/z)^2 + (a/z)^4 + (a/z)^6 + \cdots]/(2z\pi\epsilon_o)$$

(c) $V(r,\theta) = q[(a/r)^2 P_2(\cos\theta) + (a/r)^4 P_4(\cos\theta) + (a/r)^6 P_6(\cos\theta) + \cdots]/(2r\pi\epsilon_o)$

2.(a) For $r < a$, $V_{\text{in}} = Ar\cos(\theta)$
For $a < r < R$, $V_{\text{out}} = Br\cos(\theta) + C/r^2\cos(\theta)$
(b) B.C. at $r = R$: $V_{\text{out}}(R) = 0$ so that $\boxed{BR + C/R^2 = 0}$.
(c) B.C. at $r = a$: $V_{\text{in}}(a) = V_{\text{out}}(a) \Longrightarrow \boxed{Aa = Ba + C/a^2}$
$(-\partial V_{\text{out}}/\partial r)_{r=a} - (-\partial V_{\text{in}}/\partial r)_{r=a} = \sigma(\theta)/\epsilon_o \Longrightarrow \boxed{(2C/a^3 - B) - (-A) = \sigma_o/\epsilon_o}$
(d) We now solve the above 3 equations for A, B and C.
Using $C = -BR^3$ we have the two equations
$A = B(1 - R^3/a^3)$ and $-B(1 + 2R^3/a^3) + A = \sigma_o/\epsilon_o$.
This gives $B = -(\sigma_o/3\epsilon_o)a^3/R^3$, $A = (\sigma_o/3\epsilon_o)(1 - a^3/R^3)$ and $C = (\sigma_o/3\epsilon_o)a^3$
so that $V_{\text{in}} = (\sigma_o/3\epsilon_o)(1 - a^3/R^3)\, r\cos(\theta)$ which goes to 0 as $a \to R$. Why?
and $V_{\text{out}} = (\sigma_o/3\epsilon_o)(a^3/R^3)(R^3/r^2 - r)\,\cos(\theta)$ which goes to 0 as $r \to R$. Why?

3.(a) For $\rho < a$, $V_{\text{in}} = A\rho\cos(\phi)$
For $a < \rho < R$, $V_{\text{out}} = B\rho\cos(\phi) + C/\rho\cos(\phi)$
(b) B.C. at $\rho = R$: $V_{\text{out}}(R) = V_o\cos(\phi)$ so that $\boxed{BR + C/R = V_o}$.
(c) B.C. at $\rho = a$: $V_{\text{in}}(a) = V_{\text{out}}(a) \Longrightarrow \boxed{Aa = Ba + C/a}$
$\epsilon_2(-\partial V_{\text{out}}/\partial r)_{r=a} - \epsilon_1(-\partial V_{\text{in}}/\partial r)_{r=a} = \sigma_f = 0 \Longrightarrow \boxed{\epsilon_2(C/a^2 - B) - \epsilon_1(-A) = 0}$