1.(a) Dipole moment due to a ring of radius r', width dr': $\vec{dp} = 2\pi r' dr' \mathrm{P}\hat{z}$. Note that all points on this ring are the same distance to a point on the z-axis $\sqrt{r'^2 + z^2}$ and they have the same value for $(\vec{r} - \vec{r'}) \cdot \hat{z} = z$.

We then have $dV = (1/4\pi\epsilon_o)\vec{dp} \cdot (\vec{r} - \vec{r'})/|\vec{r} - \vec{r'}|^3 = (1/4\pi\epsilon_o)2\pi r' dr' \mathrm{P} z/(r'^2 + z^2)^{3/2}$.

Integrating from 0 to R, we have $V(z) = \int_{z^2}^{R^2+z^2} (1/4\pi\epsilon_o)\pi du \mathrm{P} z/u^{3/2}$ where $u = r'^2 + z^2$ so that $du = 2r'dr'$.

This gives $V(z) = (1/4\pi\epsilon_o)2\pi \mathrm{P} z \left(1/z - 1/\sqrt{R^2 + z^2}\right)$

(b) For large values of z we have $1/\sqrt{R^2 + z^2} = \left(1/\sqrt{1 + (R/z)^2}\right)/z \sim (1/z) - R^2/(2z^3) + 3R^4/(8z^5)$

so that $V(z) \sim (1/4\pi\epsilon_o)\left[\pi R^2 \mathrm{P}/z^2 - 3\pi R^4 \mathrm{P}/(4z^4)\right]$.
Note that the first term is the potential of a point dipole of magnitude $\pi R^2 \mathrm{P}$ at the origin.
(c) Generalizing to a general point (r, θ), we have $V(r, \theta) \sim (1/4\pi\epsilon_o)\left[P_1(\cos\theta)\pi R^2 \mathrm{P}/r^2 - P_3(\cos\theta)3\pi R^4 \mathrm{P}/(4r^4)\right]$.

2.(a) $\rho_B = -\nabla \cdot \vec{P} = -\nabla \cdot \hat{x} x \alpha = -\alpha$

(b) To find $\sigma_B = \vec{P} \cdot \hat{\rho}|_{\rho=R}$, we can express $\vec{P}$ in terms of the cylindrical variables:

$x = \rho\cos\phi$ and $\hat{x} = \hat{\rho}\cos\phi + \hat{\phi}\sin\phi$ so that $\vec{P} = x\hat{x}\alpha = \rho\alpha(\hat{\rho}\cos^2\phi + \hat{\phi}\cos\phi\sin\phi)$

This gives $\sigma_B = \vec{P} \cdot \hat{\rho}|_{\rho=R} = R\alpha\cos^2\phi = R\alpha(1 + cos(2\phi))/2$

(c) Bound volume charge is uniform, the E-field due to it may be found easily using the Gauss' Law:
$L2\pi\rho E_{in} = \pi\rho^2 L\rho_B/\epsilon_o \Longrightarrow E_{in} = -\alpha\rho/(2\epsilon_o)$ and $L2\pi\rho E_{out} = \pi R^2 L\rho_B/\epsilon_o \Longrightarrow E_{out} = -\alpha R^2/(2\epsilon_o\rho)$

(d) Because of the form of the bound surface charge we will have only $m = 0$ and $m = 2$ terms in the expansion with $\cos(m\phi)$ terms:

$V_{in} = A + B\rho^2\cos(2\phi)$ $\qquad$ $V_{out} = C + D\ln\rho + E\cos(2\phi)/\rho^2$.
Since an arbitrary constant may be added to the potential, let us choose $A = 0$.

The $m = 0$ boundary conditions give $0 = C + D\ln R \qquad -D/R = R\alpha/2$
The $m = 2$ boundary conditions give $BR^2 = E/R^2 \qquad 2E/R^3 + 2BR = R\alpha/2$
These result in $D = -R^2\alpha/2 \qquad C = (R^2\alpha/2)\ln R \qquad E = \alpha R^4/8 \qquad B = \alpha/8$.

$V_{in} = \alpha\rho^2\cos(2\phi)/8 \qquad V_{out} = -(R^2\alpha/2)\ln(\rho/R) + \alpha R^4\cos(2\phi)/(8\rho^2)$
Note that the logarithmic terms due to the volume and surface bound charges cancel, since the total net bound charge adds up to zero.

3.(a) Due to the selenoidal geometry, we know that the magnetic field is zero for $r > b$. If we apply Ampere's Law using a rectangular loop of length L, with one side at $r > b$ and and other side at radius r, we have $LB = \mu_o I_{enc}$. Noticing $I_{enc} = Q_{enc}/\Delta t = Q_{enc}\omega/(2\pi)$ we get

	$r < a$	$a < r < b$	$r > b$
Ampere's Law:	$LB = \mu_o\rho\pi(b^2 - a^2)L\omega/(2\pi)$	$LB = \mu_o\rho\pi(b^2 - r^2)L\omega/(2\pi)$	0
$\vec{B}$	$\hat{z}\mu_o\rho(b^2 - a^2)\omega/2$	$\hat{z}\mu_o\rho(b^2 - r^2)\omega/2$	0
$\mu_o\vec{J} = \nabla \times \vec{B}$	0	$-\hat{\phi}\frac{\partial B_z}{\partial r} = \hat{\phi}\mu_o\rho r\omega = \mu_o\rho\vec{v}$	0