1.(a) $V(z) = 2p/[4\pi\epsilon_o(z-a)^2] - 2p/[4\pi\epsilon_o(z+a)^2] = p/(2\pi\epsilon_o)\ [1/(z-a)^2 - 1/(z+a)^2]$

(b) $1/(1+\epsilon)^2 \sim 1 - 2\epsilon + 3\epsilon^2 - 4\epsilon^3 + \cdots$ and $1/(1-\epsilon)^2 \sim 1 + 2\epsilon + 3\epsilon^2 + 4\epsilon^3 + \cdots$

So $1/(1-a/z)^2 - 1/(1+a/z)^2 \sim 4(a/z) + 8(a/z)^3 + \cdots$

$V(z) = p/(2\pi\epsilon_o z^2)\ [4(a/z) + 8(a/z)^3 + \cdots]$

(c) $V(r,\theta) = 2p/(\pi\epsilon_o)\ [(a/r^3)P_2(\cos\theta) + 2(a^3/r^5)P_4(\cos\theta) + \cdots]$

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2. (a) $\vec{E}_2(\text{origin}) = 2\hat{x}p/(4\pi\epsilon_o a^3)$ so that $\vec{\tau}_1 = \vec{p}_1 \times \vec{E}_2(\text{origin}) = 2\hat{y}p^2/(4\pi\epsilon_o a^3)$

(b) $\vec{E}_1(a\hat{x}) = -\hat{z}p/(4\pi\epsilon_o a^3)$ so that $\vec{\tau}_2 = \vec{p}_2 \times \vec{E}_1(a\hat{x}) = \hat{y}p^2/(4\pi\epsilon_o a^3)$

So the total torque on the dipoles is $3\hat{y}p^2/(4\pi\epsilon_o a^3)$. By Newton's third law, the total torques and the forces on the system must add up to zero. From the fields acting on the dipoles, we can deduce that we can achieve this by forces $\vec{F}_1 = -\vec{F}_2 = -3\hat{z}p^2/(4\pi\epsilon_o a^4)$ acting on the dipoles. Let's verify:

(c)

$$\vec{F}_2 = (\vec{p}_2 \cdot \nabla)\ \vec{E}_1\Big|_{\vec{r}=a\hat{x}} = p\frac{\partial}{\partial x}\left(\frac{3pz(x\hat{x}+y\hat{y}+z\hat{z})}{(x^2+y^2+z^2)^{5/2}} - \frac{p\hat{z}}{(x^2+y^2+z^2)^{3/2}}\right)\Bigg|_{\vec{r}=a\hat{x}} \frac{1}{4\pi\epsilon_o}$$

only the second term will survive when we set $\vec{r} = a\hat{x}$, *i.e.* when $x = a$, $y = 0$ and $z = 0$:

$$\vec{F}_2 = \frac{3}{2}\frac{2xp^2\hat{z}}{(x^2+y^2+z^2)^{5/2}}\Bigg|_{\vec{r}=a\hat{x}} \frac{1}{4\pi\epsilon_o} = \frac{3p^2\hat{z}}{4\pi\epsilon_o a^4}$$

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3. (a) Form of the potential:

$$V(r,\theta) = \begin{cases} ArP_1(\cos\theta) & \text{for } r < a \\ (Br + C/r^2)P_1(\cos\theta) & \text{for } a < r < b \\ (D/r^2 - rE_o)P_1(\cos\theta) & \text{for } b < r \end{cases}$$

(We keep only terms proportional to $P_1(\cos\theta)$ because the external field is driving only that angular dependence.)

(b) Boundary conditions:

	at $r = a$	at $r = b$
Potential:	$Aa = Ba + C/a^2$	$Bb + C/b^2 = D/b^2 - bE_o$
Normal component of $\vec{D}$:	$\epsilon_o A = \epsilon(B - 2C/a^3)$	$\epsilon(B - 2c/b^3) = \epsilon_o(-2D/b^3 - E_o)$

(c) When $\epsilon = \epsilon_o$, then there is no dielectric, and one gets $V = -E_o r P_1(\cos(\theta)$ everywhere.

(d) When $\epsilon \to \infty$ the dielectric acts as a conductor and we have $V = 0$ for $r < b$.