1.(a) Let us place a surface S inside the conductor, enclosing the cavity, and apply Gauss law to it: Since the electric field inside the conductor is zero, the total enclosed charge inside the surface must be zero. So, there must be a total charge $-Q$ on the inner surface of the conductor: $\sigma_{\text{in}} = -Q/(\pi a^2)$.
We must then have a total charge $q+Q$ on the outer surface: $\sigma\text{out} = (q+Q)/(\pi b^2)$.
(b) Gauss Law implies:

$$E(r) = \begin{cases} Q/[4\pi\epsilon_o r^2] & \text{for } r < a \\ 0 & \text{for } a < r < b \text{ (inside the conductor)} \\ (Q+q)/[4\pi\epsilon_o r^2] & \text{for } r > a \end{cases}$$

(c) Spherically symmetric charge densities look like point charges from the outside. So, for $r > b$ the potential is $(Q+q)/[4\pi\epsilon_o r]$. The outer surface of the conductor (and therefore all of the conductor) will be at the potential $(Q+q)/[4\pi\epsilon_o b]$ for $a < r < b$. Finally, inside the cavity, $V(r) = \int_r^a Q/[4\pi\epsilon_o r^2]dr + V(a) = [Q/r - Q/a + (q+Q)/b]/(4\pi\epsilon_o)$ for $r < a$.

2.(a) As in the previous problem, the total charge on the surface of the cavity must be $-Q$
(b) Since the total charge of the conductor is q, the total charge on the outer surface will be $q+Q$, distributed symmetrically.
(c) Point charge will have an image $Q' = -Qr/a$ at $a' = r^2/a$. It will therefore have a force acting on it $+z$ direction with magnitude $QQ'/[(4\pi\epsilon_o)(a'-a)^2] = Q^2ar/[(4\pi\epsilon_o)(r^2-a^2)^2]$.
(d) Newton's third law indicates the conductor will have an equal but opposite force on it.
(e) From the outside, the charge density appears as spherically symmetric charge of magnitude $q+Q$, which will generate the same potential as a point charge. So the potential ar radius R will be $(q+Q)/[4\pi\epsilon_o R]$, which will be the potential of the conductor.

3.It is useful to utilize the symmetry of the structure. If the origin is placed at the center of the rectangle, we have the boundary conditions $V = 0$ at $y = \pm b/2$ and $V = V_o\cos(\pi y/b)$ at $x = \pm a/2$. So, the problem now has reflection symmetry with respect to the x and the y axes. We can therefore use functions even in x and y in the solution. Now, notice that

$$V(x,y) = V_o\cos(\pi y/b)\cosh(\pi x/b)/\cosh[\pi a/(2b)]$$

satisfies the Laplace Equation and the boundary conditions, so it must be the unique solution to the problem. If the origin is placed at the lower left corner of the rectangle, the solution will be

$$V(x,y) = V_o\sin(\pi y/b)\left(\frac{\sinh(\pi x/b)}{\sinh(\pi a/b)} + \frac{\sinh[\pi(a-x)/b]}{\sinh(\pi a/b)}\right).$$

The potential distribution looks like: