1.(a) $\rho_b = -\nabla\cdot\vec{P} = -\partial P_x/\partial x = -\alpha$.
(b) $\sigma_b = \vec{P}\cdot\hat{S} = \vec{P}\cdot(\hat{x}\cos\phi + \hat{y}\sin\phi) = \alpha x\cos\phi = \alpha R\cos^2\phi = \alpha R(1+\cos 2\phi)/2$
note that there will be only $m=0$ and $m=2$ terms in the potential.
(c) $V = A_o + A_2\rho^2\cos 2\phi$ inside and $V = B_o + B_2/\rho^2\cos 2\phi + C\ln\rho$ outside.
(Choose $A_o = 0$.) Boundary condition matching at $\rho = R$:
Potential:
$A_2R^2\cos 2\phi = B_o + B_2/R^2\cos 2\phi + C\ln R \Longrightarrow B_o + C\ln R = 0$ and $A_2R^2 = B_2/R^2$
Field:
$-(\partial V_{\text{out}}/\partial\rho - \partial V_{\text{in}}/\partial\rho)_{\rho=R} = \sigma_b/\epsilon_o$
$\Longrightarrow (2B_2/R^3 + 2A_2R)\cos 2\phi - C/R = \alpha R(1+\cos 2\phi)/2\epsilon_o$
$\Longrightarrow 2B_2/R^3 + 2A_2R = \alpha R/2\epsilon_o$ and $C/R = -\alpha R/2\epsilon_o$
we get
$C = -\alpha R^2/2\epsilon_o$, $B_o = (\alpha R^2/2\epsilon_o)\ln R$, $A_2 = \alpha/8\epsilon_o$ and $B_2 = \alpha R^4/8\epsilon_o$
so that
$V_{\text{in}} = \rho^2\alpha/8\epsilon_o\cos 2\phi$ and $V_{\text{out}} = -(\alpha R^2/2\epsilon_o)\ln(\rho/R) + (\alpha R^4/8\rho^2\epsilon_o)\cos 2\phi$
Actually, the potential due to the bound volume charge will cancel the $m=0$ term.

2.(a) Using Ampre's law around the straight wire, $2\pi yB = \mu_o I_1 \Longrightarrow B = \mu_o I_1/2\pi y$. The field will be in the $+z$ direction for $y>0$ and $-z$ direction for $y<0$. $\Longrightarrow \vec{B} = \hat{z}\mu_o I_1/2\pi y$
(b) $\vec{dl}\times\vec{B}$ has $+y$ components at all points on the loop. The x-components cancel for symmetric poins at $x\to -x$. The net force on the loop is going to be in the $+y$ direction.
(c) $\vec{F} = I_2\int\vec{dl}\times\vec{B} = I_2\int Rd\phi(-\hat{x}\sin\phi + \hat{y}\cos\phi)\times\hat{z}\mu_o I_1/(2\pi R\sin\phi)$
$= \mu_o I_1 I_2/2\pi\int(\hat{y} + \hat{x}\cos\phi/\sin\phi)d\phi = \hat{y}\mu_o I_1 I_2$ The $\hat{x}$ integral is zero due to the odd integrand, as was discussed in part (b).

---

3. (a) Applying Ampere's law for a loop inside the inner wire at radius ρ,
$B2\pi\rho = \mu_o I_{\text{enc}} = \mu_o I\pi\rho^2/\pi a^2 \Longrightarrow \vec{B}_{\text{in}} = \hat{\phi}\ \mu_o I\rho/(2\pi a^2)$
(b) For $a<\rho<b$: $B2\pi\rho = \mu_o I_{\text{enc}} = \mu_o I \Longrightarrow \vec{B}_{\text{mid}} = \hat{\phi}\ \mu_o I/(2\pi\rho)$
(c) Current is in the $\pm z$ directrion, so $\vec{A}$ is expected to be in that direction as well. This is consistent with a perpendicular magnetic field in the $\hat{\phi}$ direction. We can use the magnetic field and the relation $\oint_c\vec{A}\cdot\vec{dl} = \int\vec{B}\cdot\vec{dS}$. Choosing a loop which contains part of the z-axis (of length l) and another line segment in $-z$-direction at radius ρ, we can find the vector potential at ρ:
$-Al = \Phi_B = \int_0^a B_{\text{in}}l d\rho + \int_a^\rho B_{\text{mid}}l d\rho = (\mu_o Il/2\pi)(\int_0^a\rho/a^2 d\rho + \int_a^\rho d\rho/\rho)$
$\Longrightarrow \vec{A} = -\hat{z}(\mu_o I/2\pi)[1/2 + \ln(\rho/R)]$
(Note the - sign due to the application of right-hand rule for the contour C and the surface S.)
(d) Since we only have the A_z component, the relative terms in the curl are:
$\nabla\times\vec{A}(\rho,\phi,z) = \hat{\rho}\ (1/\rho)\ \partial A_z/\partial\phi - \hat{\phi}\ \partial A_z/\partial\rho = \hat{\phi}\ \mu_o I/2\pi\rho$ as consistent with part (b).